

Discussion 9

1. Show that for any (nonempty) sets A, B, C and any functions $f : A \rightarrow B$ and $g : B \rightarrow C$, if f and g are onto, then $g \circ f$ is also onto.

Use this lemma to prove that if $|A| \geq |B|$ and $|B| \geq |C|$ then $|A| \geq |C|$.

2. Prove that if A, B, C, D are sets with $|A| = |B|$ and $|C| = |D|$ then $|A \times C| = |B \times D|$.

3. Give an example of two uncountable sets A and B such that:

(a) $A - B$ is finite.

(b) $A - B$ is countably infinite.

(c) $A - B$ is uncountable.

(d) $A \cap B$ is finite.

(e) $A \cap B$ is countably infinite.

(f) $A \cap B$ is uncountable.