Discussion 8

1. Is each of the following sets **finite** or **infinite**?

(a) The negative integers

(b) The even integers

(c) The integers less than 100

(d) The real numbers between 0 and $\frac{1}{2}$

(e) The positive integers less than 1000000

(f) The integers that are multiples of 7

(g) The integers that are divisors of 2017

(h) The set $\{2, 3\} \times \mathbb{Z}^+$

(i) The set $\{2, 3\} \cap \mathbb{Z}^+$

(j) The set $\{2, 3\} \cup \mathbb{Z}^+$

2. Show that for any (nonempty) sets A, B, C and any functions $f : A \to B$ and $g : B \to C$, if f and g are one-to-one, then $g \circ f$ is also one-to-one.
Use this lemma to prove that if $|A| \leq |B|$ and $|B| \leq |C|$ then $|A| \leq |C|$.

3. In Cantor's diagonalization argument, we show that any function from a set to its power set is not a bijection. To get a better feel for this argument, show why each of the following specific functions are not bijections. *Is each function one-to-one? Is each function onto?*

(a) $f : \{a, b, c\} \to \mathcal{P}(\{a, b, c\})$ given by

x	$f(x)$
a	$\emptyset$
b	$\{a, b, c\}$
c	$\{a, b\}$

(b) $f : \mathbb{N} \to \mathcal{P}(\mathbb{N})$ given by $f(n) = \{0, 1, \ldots, n\}$.

(c) $f : \{0, 1\}^* \to \mathcal{P}(\{0, 1\}^*)$ given recursively by $f(\lambda) = \emptyset$ and $f(wx) = f(w) \cup \{x\}$