Discussion 7

1. *(cf. Rosen 5.3 Exercise 39)* When does a string belong to the set A of bit strings defined recursively by

$$\lambda \in A$$
$$0x1 \in A \qquad \text{if } x \in A$$

(where λ is the empty string)?

2. *(cf. Rosen 5.3 Exercises 34-35)* The **reversal** of a string is the string consisting of the symbols of the string in reverse order. The reversal of the string w is denoted by w^R.

(a) Find the reversal of the following bit strings

0101

11011

100010010111

(b) Give a recursive definition of the reversal of a bit string.
Note: this is a recursive definition of a function whose input is a string and whose output is its reversal.

3. *(cf. Rosen 5.3 Exercises 37, 41)* Give a recursive definition of w^i, where w is a bit string and i is a nonnegative integer. (Here, w^i represents the concatenation of i copies of the string w.)

Use mathematical induction to show that, for all nonnegative integers i and all bit strings w,

$$l(w^i) = i \cdot l(w)$$

Recall: The definition of the length of a bit string is given recursively by

$$l(\lambda) = 0$$
$$l(wx) = l(w) + 1 \qquad \text{if } w \text{ is a bit string and } x \in \{0, 1\}$$

You may use the result (example 12 in Sec 5.3) that $l(x + y) = l(x) + l(y)$ for any bit strings x and y. Note: the proof of this result is a good example of structural induction.