Discussion 7

1. (cf. Rosen 5.3 Exercise 39) When does a string belong to the set $A$ of bit strings defined recursively by

$$
\begin{align*}
\lambda & \in A \\
0x1 & \in A \quad \text{if } x \in A
\end{align*}
$$

(where $\lambda$ is the empty string)?

2. (cf. Rosen 5.3 Exercises 34-35) The **reversal** of a string is the string consisting of the symbols of the string in reverse order. The reversal of the string $w$ is denoted by $w^R$.

(a) Find the reversal of the following bit strings

0101

11011

100010010111

(b) Give a recursive definition of the reversal of a bit string.

*Note: this is a recursive definition of a function whose input is a string and whose output is its reversal.*
3. (cf. Rosen 5.3 Exercises 37, 41) Give a recursive definition of \( w^i \), where \( w \) is a bit string and \( i \) is a nonnegative integer. (Here, \( w^i \) represents the concatenation of \( i \) copies of the string \( w \).)

Use mathematical induction to show that, for all nonnegative integers \( i \) and all bit strings \( w \),

\[
l(w^i) = i \cdot l(w)
\]

Recall: The definition of the length of a bit string is given recursively by

\[
\begin{align*}
l(\lambda) &= 0 \\
l(wx) &= l(w) + 1 & \text{if } w \text{ is a bit string and } x \in \{0, 1\}
\end{align*}
\]

You may use the result (example 12 in Sec 5.3) that \( l(x + y) = l(x) + l(y) \) for any bit strings \( x \) and \( y \).

Note: the proof of this result is a good example of structural induction.