Discussion 5

We use the following conventions, following the book on page 116 (section 2.1)

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ the set of integers

$\mathbb{Z}^+ = \{1, 2, 3 \ldots\}$ the set of positive integers

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$, the set of natural numbers

$\mathbb{Q} = \{\frac{p}{q} \mid p, q$ integers and $q \neq 0\}$

$\mathbb{R}$, the set of real numbers

$\mathbb{R}^+$, the set of positive real numbers

1. Write each of these sets without using set builder notation, i.e. as an explicit collection of elements.

(a) $\{x \mid x$ is a real number such that $x^2 = 1\}$

(b) $\{x \mid x$ is a positive integer less than $12\}$

(c) $\{x \mid x$ is the square of an integer and $x < 100\}$

(d) $\{x \mid x$ is an integer such that $x^2 = 2\}$

(e) List all pairs of the sets in (a), (b), (c), (d) which share at least one member.

(f) List all pairs of the sets in (a), (b), (c), (d) which share no members.

(cf. Rosen 2.1 Exercise 1)

2. Prove or disprove each of the following statements.

(a) There is no irrational integer.

(b) Every positive integer is a natural number.

(c) No number is both real and irrational.

(d) Every perfect square is a positive integer.