Discussion 4

1. What is wrong with this argument?

Let $S(x, y)$ be “$x$ is shorter than $y$.” Given the premise $\exists s S(s, \text{Max})$, it follows that $S(\text{Max}, \text{Max})$. Then by existential generalization it follows that $\exists x S(x, x)$ so that someone is shorter than him- or herself.

(cf. Rosen 1.6 Exercise 18)

2. Use a direct proof to show that every odd integer is the difference of two squares.

(cf. Rosen 1.7 Exercise 7)
3. Prove that $\sqrt{3}$ is irrational.