

Discussion 2

1. In this question, you will construct a circuit that takes a pair of two-bit integers $(x_1x_0)_2$ and $(y_1y_0)_2$ and computes the **three** output bits for their integer **sum**.

For example, the sum of $(10)_2$ and $(11)_2$ is $(101)_2$ and could be computed by long sum as

$$\begin{array}{r}
 10 \quad \text{This is } x_1x_0 \\
 + 11 \quad \text{This is } y_1y_0 \\
 \hline
 101 \quad \text{This is the sum.}
 \end{array}$$

- a. Use our work in class on adding integers in binary to express each of the three bits of output as compound propositions with inputs x_0, x_1, y_0, y_1 .

Note: the compound proposition may include the operators $\neg, \vee, \wedge, \oplus$.

- b. Draw a logic circuit with four inputs and three outputs implementing your design in part b.

Note: the logic circuit may include the gates for AND, OR, and XOR, and each gate may have as many inputs as you need.

2. Express each of these system specifications using the propositions p “The user enters a valid password”, q “Access is granted”, and r “The user has paid the subscription fee”.

“If the user has paid the subscription fee but does not enter a valid password, then access is not granted.”

“Access is granted whenever the user has paid the subscription fee and enters a valid password.”

“Access is denied if the user has not paid the subscription fee.”

Are these system specifications consistent?