Midterm Review

Image Processing
CSE 166
Lecture 10
Topics covered

• Image acquisition, geometric transformations, and image interpolation
• Intensity transformations
• Spatial filtering
• Fourier transform and filtering in the frequency domain
• Image restoration
• Color image processing
Image acquisition
## Geometric transformations

<table>
<thead>
<tr>
<th>Transformation Name</th>
<th>Affine Matrix, $A$</th>
<th>Coordinate Equations</th>
<th>Example</th>
</tr>
</thead>
</table>
| Identity                            | \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] | $x' = x$  
$y' = y$ | T         |
| Scaling/Reflection (For reflection, set one scaling factor to $-1$ and the other to $0$) | \[
\begin{bmatrix}
c_x & 0 & 0 \\
0 & c_y & 0 \\
0 & 0 & 1
\end{bmatrix}
\] | $x' = c_x x$  
$y' = c_y y$ | T         |
| Rotation (about the origin)        | \[
\begin{bmatrix}
cos \theta & -sin \theta & 0 \\
sin \theta & cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\] | $x' = x \cos \theta - y \sin \theta$  
$y' = x \sin \theta + y \cos \theta$ | T         |
| Translation                        | \[
\begin{bmatrix}
1 & 0 & t_x \\
0 & 1 & t_y \\
0 & 0 & 1
\end{bmatrix}
\] | $x' = x + t_x$  
$y' = y + t_y$ | T         |
| Shear (vertical)                   | \[
\begin{bmatrix}
1 & \delta_x & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] | $x' = x + \delta_x y$  
$y' = y$ | T         |
| Shear (horizontal)                 | \[
\begin{bmatrix}
1 & 0 & 0 \\
\delta_h & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] | $x' = x$  
$y' = \delta_h x + y$ | T         |
Intensity transformations

Contrast stretching function

Thresholding function
Intensity transformations

Some basic transformation functions
Gamma transformation

FIGURE 3.6
Plots of the gamma equation $s = cr^\gamma$ for various values of $\gamma$ ($c = 1$ in all cases). Each curve was scaled independently so that all curves would fit in the same graph. Our interest here is on the shapes of the curves, not on their relative values.
Gamma transformation

Dark image

$\gamma < 1$

FIGURE 3.8
(a) Magnetic resonance image (MRI) of a fractured human spine (the region of the fracture is enclosed by the circle).

(b)–(d) Results of applying the transformation in Eq. (3-5) with $c = 1$ and $\gamma = 0.6, 0.4$, and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)
Gamma transformation

Light image

$\gamma > 1$

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3-5) with $\gamma = 3.0$, 4.0, and 5.0, respectively. ($c = 1$ in all cases.) (Original image courtesy of NASA.)
Piecewise-linear transformations

- Contrast stretching
- Intensity-level slicing
- Bit-plane slicing
Contrast stretching

**FIGURE 3.10**
Contrast stretching.
(a) Piecewise linear transformation function. (b) A low-contrast electron microscope image of pollen, magnified 700 times. (c) Result of contrast stretching. (d) Result of thresholding.
(Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)
Intensity-level slicing

**Figure 3.11**
(a) This transformation function highlights range \([A, B]\) and reduces all other intensities to a lower level.
(b) This function highlights range \([A, B]\) and leaves other intensities unchanged.

**Figure 3.12**
(a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected range set near black, so that the grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)
Bit-plane slicing

**Figure 3.13**
Bit-planes of an 8-bit image.

**Figure 3.14**
(a) An 8-bit grayscale image of size 837 × 988 pixels.
(b) through (i) Bit planes 8 through 1, respectively, where plane 1 contains the least significant bit. Each bit plane is a binary image. Figure (a) is an SEM image of a trophozoite that causes a disease called *giardiasis.* (Courtesy of Dr. Stan Erlandsen, U.S. Center for Disease Control and Prevention.)
Histogram

Similar to probability density function (pdf)
Histogram equalization

**Figure 3.18** (a) An arbitrary PDF. (b) Result of applying Eq. (3-11) to the input PDF. The resulting PDF is always uniform, independently of the shape of the input.
Histogram equalization

**Figure 3.20** Left column: Images from Fig. 3.16. Center column: Corresponding histogram-equalized images. Right column: histograms of the images in the center column (compare with the histograms in Fig. 3.16).
Histogram equalization

**FIGURE 3.22**
(a) Image from Phoenix Lander.
(b) Result of histogram equalization.
(c) Histogram of image (a).
(d) Histogram of image (b).
(Original image courtesy of NASA.)
Spatial filtering (2D)
Correlation and convolution (2D)

**Figure 3.36**
Correlation (middle row) and convolution (last row) of a 2-D kernel with an image consisting of a discrete unit impulse. The 0’s are shown in gray to simplify visual analysis. Note that correlation and convolution are functions of $x$ and $y$. As these variable change, they displace one function with respect to the other. See the discussion of Eqs. (3.45) and (3.46) regarding full correlation and convolution.

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<th>Convolution result</th>
<th>Full convolution result</th>
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Smoothing kernels

Average (box kernel)

Weighted average (Gaussian kernel)
Smoothing with box kernel

Input image

3x3
11x11
21x21
Smoothing with Gaussian kernel

**Figure 3.41**
(a) Sampling a Gaussian function to obtain a discrete Gaussian kernel. The values shown are for \( K = 1 \) and \( \sigma = 1 \). (b) Resulting \( 3 \times 3 \) kernel [this is the same as Fig. 3.37(b)].

<table>
<thead>
<tr>
<th>Standard deviation ( \sigma )</th>
<th>Percent of total volume under surface</th>
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<tbody>
<tr>
<td>1</td>
<td>39.35</td>
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<tr>
<td>2</td>
<td>86.47</td>
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<tr>
<td>3</td>
<td>98.89</td>
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</table>

Volume under surface greater than \( 3\sigma \) is negligible
Smoothing with Gaussian kernel

Input image

σ = 3.5
21x21

σ = 7
43x43
Derivatives

Figure 3.50
(a) A section of a horizontal scan line from an image, showing ramp and step edges, as well as constant segments.
(b) Values of the scan line and its derivatives.
(c) Plot of the derivatives, showing a zero crossing. In (a) and (c) points were joined by dashed lines as a visual aid.
Sharpening filters

**FIGURE 3.52**
(a) Blurred image of the North Pole of the moon.
(b) Laplacian image obtained using the kernel in Fig. 3.51(a).
(c) Image sharpened using Eq. (3-63) with $c = -1$.
(d) Image sharpened using the same procedure, but with the kernel in Fig. 3.51(b). (Original image courtesy of NASA.)

**FIGURE 3.53**
The Laplacian image from Fig. 3.52(b), scaled to the full $[0, 255]$ range of intensity values. Black pixels correspond to the most negative value in the unscaled Laplacian image, grays are intermediate values, and white pixels correspond to the highest positive value.
Review

• Complex numbers
Overview: Image processing in the frequency domain

Image in spatial domain $f(x,y)$ \rightarrow \text{Fourier transform} \rightarrow \text{Image in frequency domain $F(u,v)$}

Image in spatial domain $g(x,y)$ \leftarrow \text{Inverse Fourier transform} \leftarrow \text{Image in frequency domain $G(u,v)$}

Jean-Baptiste Joseph Fourier
1768-1830
1D Fourier series

Sines and cosines

Periodic function

Weighted by magnitude

Shifted by phase
Sampling

- A continuous function.
- Train of impulses used to model sampling.
- Sampled function formed as the product of (a) and (b).
- Sample values obtained by integration and using the sifting property of impulses. (The dashed line in (c) is shown for reference. It is not part of the data.)
Sampling

Fourier transform of function

Fourier transforms of sampled function

Over-sampled

Critically-sampled

Under-sampled
The sampling theorem

Fourier transform of function

Fourier transform of sampled function

Critically-sampled
Recovering $F(\mu)$ from $\tilde{F}(\mu)$

Fourier transform of sampled function

Over-sampled

Ideal lowpass filter

Product of above

Recovered
Aliasing

Continuous

Discrete

Under-sampled

Different

Over-sampled

Sampled at same rate

Alias: a false identity

Identical

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Fourier transform of sampled function and extracting one period

Over-sampled

Under-sampled

1D

Recovered

2D

Imperfect recovery due to interference

Footprint of a 2-D ideal lowpass (box) filter

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Centering the DFT

In MATLAB, use `fftshift` and `ifftshift`
Centering the DFT

Original DFT (look at corners)

Shifted DFT

DFT (look at corners)

Log of shifted DFT
Contributions of magnitude and phase to image formation

IDFT: Phase only (zero magnitude)

IDFT: Magnitude only (zero phase)

IDFT: Boy magnitude and rectangle phase

IDFT: Rectangle magnitude and boy phase
Filtering using convolution theorem

Filtering in spatial domain using convolution

Filtering in frequency domain using product without zero-padding

Expected result

Wraparound error
Filtering using convolution theorem

Filtering in frequency domain using product with zero-padding

Zero padding

Fourier transform

Product

Gaussian lowpass filter in frequency domain

Inverse Fourier transform

no wraparound error
Filtering in the frequency domain

- Ideal lowpass filter (LPF)
  - Frequency domain
Filtering in the frequency domain

- Ideal lowpass filter (LPF)
  - Spatial domain

\[ H(u,v) \quad h(x,y) \]
Filtering in the frequency domain

• Gaussian lowpass filter (LPF)
Filtering in the frequency domain

- Butterworth lowpass filter (LPF)
Filtering in the frequency domain

Ideal LPF

Gaussian LPF

Butterworth LPF
Highpass filter (HPF) Frequency domain

- Ideal HPF
- Gaussian HPF
- Butterworth HPF
Highpass filter (HPF)  
Spatial domain

Ideal HPF  Gaussian HPF  Butterworth HPF
Filtering in the frequency domain

Ideal HPF  Gaussian HPF  Butterworth HPF
Filtering in the frequency domain

Frequency domain

Spatial domain

Lowpass filter

Sharpening filter
Filtering in the frequency domain

Lowpass filter  Highpass filter  Offset highpass filter

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Bandreject filters

Ideal

Gaussian

Butterworth

\[ H(u, v) \]

\[ H(u, v) \]

\[ H(u, v) \]
Model of image degradation, then restoration
Histograms of sample patches

Sample “flat” patches from images with noise

Identify closest probability density function (pdf) match:

- Gaussian
- Rayleigh
- Uniform
Mean filters

X-ray image

Additive Gaussian noise

Arithmetic mean filtered

Geometric mean filtered
Order-statistic filters

Additive salt and pepper noise

2x median filtered

1x median filtered

3x median filtered
Comparing filters

- Additive uniform + salt and pepper noise
- Arithmetic mean filtered
- Median filtered
- Geometric mean filtered
- Alpha-trimmed mean filtered
Adaptive filters

Additive Gaussian noise

Geometric mean filtered

Arithmetic mean filtered

Adaptive noise reduction filtered
Periodic noise

Additive sinusoidal noise

Conjugate impulses

DFT magnitude
Notch reject filters
Notch reject filter

Degraded image

Filter in frequency domain

DFT magnitude

Conjugate impulses

Conjugate impulses

Estimate of original image
Estimation of degradation function by experimentation

Impulse of light

Imaged (degraded) impulse
Estimation of degradation function by mathematical modeling

Atmospheric turbulence model
Image restoration

Inverse filtering
RGB color model

RGB color cube

RGB coordinates
HSI color model: Relationship to RGB color model

RGB color cube rotated such that line joining black and white (intensity axis) is vertical

All colors with cyan hue
HSI color model

RGB color cube rotated such that observer is on intensity axis, beyond white looking towards black

HSI intensity axis

Shape does not matter, only angle from red
Color models

CMYK

CMY

RGB

HSI

Full color image

Cyan

Magenta

Yellow

Black

Cyan

Magenta

Yellow

Red

Green

Blue

Hue

Saturation

Intensity

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Intensity slicing

Grayscale to 2 colors
Intensity slicing

Grayscale to 2 colors
Intensity slicing

Grayscale to 256 colors
Intensity to color transformations

Grayscale input image

RGB output image
Intensity to color transformations

X-ray grayscale input image

Without explosive

With explosive

RGB output images

Misses explosive
Intensity to color transformations

Multiple grayscale input images

Single RGB output image
Intensity to color transformations

Red (R)  Green (G)  Blue (B)

Multiple satellite grayscale input images

Near infrared (NIR)  NIR,G,B as RGB  R,NIR,B as RGB

Output RGB images
Full-color image processing

Spatial filtering: process each channel independently

Gray-scale image

RGB color image

Spatial mask

(x, y)
Full-color image processing

Spatial filtering: image smoothing

All RGB channels

HSI intensity channel only

Difference
Full-color image processing

Spatial filtering: image sharpening

All RGB channels

HSI intensity channel only

Difference
Full-color image processing

Histogram equalization: **do not** process each channel independently

1. RGB to HSI
2. Histogram equalize HSI intensity
3. HSI to RGB
4. HSI saturation adjustment