1. We can’t recommend everybody the same thing (even if they all want it!)

- So far, we have an algorithm that takes “budgets” into account, so that users are shown a limited number of ads, and ads are shown to a limited number of users.
- **But**, all of this only applies if we see all the users and all the ads *in advance*

- This is what’s called an **offline algorithm**
On Monday we looked at matching problems which are a flexible way to find compatible user-to-item matches, while also enforcing “budget” constraints.
2. We need to be **timely**

- But in many settings, users/queries come in one at a time, and need to be shown some (highly compatible) ads
- But we still want to satisfy the same quality and budget constraints

- So, we need **online algorithms** for ad recommendation
What is adwords?

**Adwords** allows advertisers to bid on keywords

- This is similar to our matching setting in that advertisers have limited **budgets**, and we have limited space to show ads

![Image from blog.adstage.io](image from blog.adstage.io)
What is adwords?

**Adwords** allows advertisers to bid on keywords

- This is similar to our matching setting in that advertisers have limited **budgets**, and we have limited space to show ads
  - **But,** it has a number of key differences:

  1. Advertisers don’t pay for impressions, but rather they pay when their ads get clicked on
  2. We don’t get to see all of the queries (keywords) in advance – **they come one-at-a-time**
Adwords allows advertisers to bid on keywords

- We still want to match advertisers to keywords to satisfy budget constraints
- But can’t treat it as a monolithic optimization problem like we did before
- Rather, we need an online algorithm
What is adwords?

Suppose we’re given

- Bids that each advertiser is willing to make for each query
  \[ f(q, a) \]
  (this is how much they’ll pay \textbf{if the ad is clicked on})
- Each is associated with a click-through rate
  \[ \text{ctr}(q, a) \]
  \[ c(q, a) \times f(q, a) \]
- Budget for each advertiser \[ b(a) \] (say for a 1-week period)
- A limit on how many ads can be returned for each query
What is adwords?

And, every time we see a query

- Return at most the number of ads that can fit on a page
- And which won’t overrun the budget of the advertiser (if the ad is clicked on)

Ultimately, what we want is an algorithm that maximizes revenue – the number of ads that are clicked on, multiplied by the bids on those ads.
What we’d like is:

the revenue should be as close as possible to what we would have obtained if we’d seen the whole problem up front
(i.e., if we didn’t have to solve it online)

We’ll define the **competitive ratio** as:

\[
\frac{\text{revenue of our algorithm}}{\text{revenue of an optimal algorithm}}
\]

Greedy solution

Let’s start with a simple version of the problem...

1. One ad per query
2. Every advertiser has the same budget
3. Every ad has the same click through rate
4. All bids are either 0 or 1
   (either the advertiser wants the query, or they don’t)
Greedy solution

Then the greedy solution is...

• Every time a new query comes in, select any advertiser who has bid on that query (who has budget remaining)

• What is the competitive ratio of this algorithm?
### Greedy solution

<table>
<thead>
<tr>
<th>Advertisers</th>
<th>Budget</th>
<th>Bids</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$2</td>
<td>„z“</td>
</tr>
<tr>
<td>B</td>
<td>$2</td>
<td>„y“</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Query</th>
<th>B</th>
<th>B</th>
<th>x</th>
<th>y</th>
<th>y</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>„Greedy“</td>
<td>B</td>
<td>B</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>$2</td>
</tr>
<tr>
<td>„Opt“</td>
<td>A</td>
<td>A</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>$4</td>
</tr>
</tbody>
</table>

**Comparison ratio:** \[rac{\$2}{\$4} = \frac{1}{2}\]
The balance algorithm

A better algorithm...

- Every time a new query comes in, amongst advertisers who have bid on this query, select the one with the largest remaining budget

- How would this do on the same sequence?

```
query:  "greedy" 
D(5),  A(3),  B,  x,  y,  y 
A, A, A, B, B 
profit:  $2  $4
C.R. = 3/4
```
The balance algorithm

A better algorithm...

• Every time a new query comes in, amongst advertisers who have bid on this query, select the one with the largest remaining budget

• In fact, the competitive ratio of this algorithm (still with equal budgets and fixed bids) is \((1 - 1/e) \sim 0.63\)

The balance algorithm

What if bids aren’t equal?

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Bid (on q)</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>110</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>

"Greedy" A A A A A A A A A A → $60
Opt: B D B D D D B B B B B B → $100

c.r = 1/10
The balance algorithm

What if bids aren’t equal?

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Bid (on q)</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>10100</td>
</tr>
<tr>
<td>B</td>
<td>100</td>
<td>10660</td>
</tr>
</tbody>
</table>

\[999 \ldots \times 100 \ldots 9\]

\[AAA \ldots \times 100 \ldots A \Rightarrow \$100\]

\[BBB \ldots \times 100 \ldots B \Rightarrow \$1600\]

\[c.5 \times \frac{1}{100} \Rightarrow c.c = 0\]
The balance algorithm v2

We need to make two modifications

• We need to consider the bid amount when selecting the advertiser, and bias our selection toward higher bids
• We also want to use some of each advertiser’s budget (so that we don’t just ignore advertisers whose budget is small)
The balance algorithm v2

Advertiser: $A_i$

**fraction** of budget remaining: $f_i \in [0, 1]$

bid on query $q$: $x_i(q) \in \mathbb{R}^+$

Assign queries to whichever advertiser maximizes:

$$\Psi_i(q) = x_i(q) \cdot (1 - e^{-f_i})$$

(could multiply by click-through rate if click-through rates are not equal)
The balance algorithm v2

Properties

• This algorithm has a competitive ratio of $(1 - \frac{1}{e})$.

• In fact, there is no online algorithm for the adwords problem with a competitive ratio better than $(1 - \frac{1}{e})$.

(proof is too deep for me...)
So far we have seen...

• An online algorithm to match advertisers to users (really to queries) that handles both bids and budgets
• We wanted our online algorithm to be as good as the offline algorithm would be – we measured this using the competitive ratio
• Using a specific scheme that favored high bids while trying to balance the budgets of all advertisers, we achieved a ratio of \((1 - \frac{1}{e})\).
• And no better online algorithm exists!
We haven’t seen...

- AdWords actually uses a *second-price* auction (the winning advertiser pays the amount that the second highest bidder bid)
- Advertisers don’t bid on specific queries, but inexact matches (‘broad matching’) – i.e., queries that include subsets, supersets, or synonyms of the keywords being bid on
Further reading:

- Mining of Massive Datasets – “The Adwords Problem”
- AdWords and Generalized On-line Matching (A. Mehta)
So far...

1. We’ve seen algorithms to handle **budgets** between users (or queries) and advertisers
2. We’ve seen an **online** version of these algorithms, where queries show up one at a time
3. Next, how can we **learn** about which ads the user is likely to click on in the first place?
3. How can we learn about which ads the user is likely to click on in the first place?

- If we see the user click on a car ad once, we know that (maybe) they have an interest in cars
- So... we know they like car ads, should we keep recommending them car ads?

- No, they’ll become less and less likely to click it, and in the meantime we won’t learn anything new about what else the user might like
Bandit algorithms

- **Sometimes** we should surface car ads (which we know the user likes),
- **but sometimes**, we should be willing to take a risk, so as to learn what **else** the user might like
**Setup**

At each round $t$, we select an arm to pull.

- We’d like to pull the arm to maximize our total reward.

### $K$ Bandits (i.e., $K$ Arms)

<table>
<thead>
<tr>
<th>round $t$</th>
<th>$t = 1$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 reward</td>
<td>1 0 0 1 1 0 1</td>
<td>0 0 1</td>
<td>1 0 1 0</td>
<td>1 1 1 0 1 1 0</td>
<td>1 0 1 0 0 0 0 0</td>
<td>0 1 0 0 0 1 0 0</td>
<td>0 0 0 0 0 1 1 0</td>
<td>0 0 1 0 0 0 0 1</td>
<td>1 0 1 0 0 0 0 1</td>
</tr>
</tbody>
</table>

- Reward $g_{k,t}$
Setup

K bandits (i.e., $K$ arms)

- At each round $t$, we select an arm to pull
- We’d like to pull the arm to maximize our total reward
- **But** – we don’t get to see the reward function!

![Diagram showing $K$ bandits with rewards $g_{k,t}$ for each round $t$.]
Setup

• At each round $t$, we select an arm to pull
• We’d like to pull the arm to maximize our total reward
• **But** – we don’t get to see the reward function!
• All we get to see is the reward we got for the arm we picked at each round

**$K$ bandits (i.e., $K$ arms)**

- **round $t$**
  - $t = 1$
    - $1 \ ? \ ? \ ? \ ? \ ? \ ? \ ?$
  - $2$
    - $? \ 0 \ ? \ ? \ ? \ ? \ ?$
  - $3$
    - $? \ ? \ ? \ ? \ ? \ 1 \ ? \ ?$
  - $4$
    - $? \ ? \ ? \ ? \ ? \ 0 \ ? \ ?$
  - $5$
    - $0 \ ? \ ? \ ? \ ? \ ? \ ? \ ?$
  - $6$
    - $? \ ? \ ? \ ? \ ? \ 0 \ ? \ ?$
  - $7$
    - $? \ ? \ ? \ ? \ ? \ ? \ ? \ 1$
  - $8$
    - $? \ ? \ ? \ ? \ ? \ ? \ ? \ 1$
  - $9$
    - $? \ ? \ ? \ ? \ ? \ ? \ ? \ 1$

reward $g_{k,t}$
Setup

\[ K : \text{number of arms (ads)} \]
\[ n : \text{number of rounds} \]
\[ g_t = (g_{1,t}, \ldots, g_{K,t}) \in [0, 1]^K : \text{rewards} \]
\[ l_t \in \{1, \ldots, K\} : \text{which arm we pick at each round} \]
\[ g_{l_t,t} \in [0, 1] : \text{how much (0 or 1) this choice wins us} \]

Want to minimize **regret:**

\[ R_n = (\max_{i=1 \ldots K} \mathbb{E} \sum_{t=1}^{n} g_{i,t}) - \mathbb{E} \sum_{t=1}^{n} g_{l_t,t} \]

- reward we **could** have got, if we had played optimally
- reward our strategy would get (in expectation)
Goal

• We need to come up with a **strategy** for selecting arms to pull (ads to show) that would maximize our expected reward

• For the moment, we’re assuming that rewards are static, i.e., that they don’t change over time
Strategy 1 – “epsilon first”

• Pull arms at random for a while to learn the distribution, then just pick the best arm
• (show random ads for a while until we learn the user’s preferences, then just show what we know they like)

\[ \epsilon \cdot n \quad : \text{Number of steps to sample randomly} \]
\[ (1 - \epsilon) \cdot n \quad : \text{Number of steps to choose optimally} \]
Strategy 1 – “epsilon first”

- Pull arms at random for a while to learn the distribution, then just pick the best arm
- (show random ads for a while until we learn the user’s preferences, then just show what we know they like)

\[ \text{best } k = \arg \max_k \]
Strategy 2 – “epsilon greedy”

- Select the best lever most of the time, pull a random lever some of the time
- (show random ads sometimes, and the best ad most of the time)

\[ \epsilon \] : Fraction of times to sample randomly
\[ (1 - \epsilon) \] : Fraction of times to choose optimally

- Empirically, worse than epsilon-first
- Still doesn’t handle context/time
Strategy 3 – “epsilon decreasing”

• Same as epsilon-greedy (Strategy 2), but epsilon decreases over time
Strategy 4 – “Adaptive epsilon greedy”

- Similar to epsilon-decreasing (Strategy 3), but epsilon can increase and decrease over time.
Extensions

• The reward function may not be static, i.e., it may change each round according to some process
• It could be chosen by an adversary
• The reward may not be [0,1] (e.g. clicked/not clicked), but instead a could be a real number (e.g. revenue), and we’d want to estimate the distribution over rewards
Extensions – **Contextual** Bandits

- There could be **context** associated with each time step
  - The query the user typed
  - What the user saw during the **previous** time step
  - What other actions the user has recently performed
  - Etc.

\[ q_{k_t} = \phi(t) \cdot 0 \]
Applications (besides advertising)

- Clinical trials
  (assign drugs to patients, given uncertainty about the outcome of each drug)
- Resource allocation
  (assign person-power to projects, given uncertainty about the reward that different projects will result in)
- Portfolio design
  (invest in ventures, given uncertainty about which will succeed)
- Adaptive network routing
  (route packets, without knowing the delay unless you send the packet)
Questions?

Further reading:
Tutorial on Bandits:
https://sites.google.com/site/banditstutorial/
CSE 158 – Lecture 14
Web Mining and Recommender Systems

Case study – Turning down the noise
“Turning down the noise in the Blogosphere”
(By Khalid El-Arini, Gaurav Veda, Dafna Shahaf, Carlos Guestrin)

**Goals:**

1. Help to **filter** huge amounts of content, so that users see content that is **relevant** – rather than seeing popular content over and over again
2. Maximize **coverage** so that a variety of different content is recommended
3. Make recommendations that are **personalized** to each user

some slides http://www.select.cs.cmu.edu/publications/paperdir/kdd2009-elarini-veda-shahaf-guestrin.pptx
“Turning down the noise in the Blogosphere”

(By Khalid El-Arini, Gaurav Veda, Dafna Shahaf, Carlos Guestrin)

Similar to our goals with bandit algorithms

1. Help to filter huge amounts of content, so that users see content that is relevant—rather than seeing popular content over and over again
2. Maximize coverage so that a variety of different content is recommended
3. Make recommendations that are personalized to each user

- **Exploit** by recommending content that we believe the user is likely to enjoy (personalization)
- **Explore** by recommending a variety of content (coverage)
Turning down the noise

1. Help to **filter** huge amounts of content, so that users see content that is **relevant**

Turning down the noise

2. Maximize coverage so that a variety of different content is recommended
3. Make recommendations that are personalized to each user.
1. Data and problem setting

- **Data:** Blogs ("the blogosphere")

- **Comparison:** other systems that aggregate blog data
1. Data and problem setting

• **Low-level features:**
  Bags-of-words (week 6/7), noun phrases, named entities

• **High-level features:**
  Low-dimensional document representations, topic models (week 3, week 7)
2. Maximize coverage

- We’d like to choose a (small) set of documents that maximally cover the set of features the user is interested in (later)
2. Maximize **coverage**

![Image of features and posts]

\[ F(A) = \sum_{f \in U} w_f \cdot \text{cover}_A(f) \]

- Can be done (approximately) by selecting documents greedily (with an approximation ratio of \(1 - 1/e\))
2. Maximize coverage

Works pretty well! (and there are some comparisons to existing blog aggregators in the paper)

But – no personalization
3. Personalize

\[ F(\mathcal{A}) = \sum_{f \in \mathcal{U}} \pi_{u,f} \cdot w_f \cdot \text{cover}_{\mathcal{A}}(f) \]

- Need to learn weights for each user based on their feedback (e.g. click/not-click) on each post
3. Personalize

\[ F(\mathcal{A}) = \sum_{f \in \mathcal{U}} \pi_{u,f} \cdot w_f \cdot \text{cover}_A(f) \]

- Need to learn weights for each user based on their feedback (e.g. click/not-click) on each post
  - A click (or thumbs-up) on a post increases \( \pi_{u,f} \) for the features \( f \) associated with the post
  - Not clicking (or thumbs-down) decreases \( \pi_{u,f} \) for the features \( f \) associated with the post
3. Personalize

feedback on articles suggested

weighted interest in topic

day 1  day 2  day 3
• Want an algorithm that **covers** the set of topics that each user wants to see
• Articles can be chosen **greedily**, while still covering the topics nearly optimally
• The topics to cover can also be **personalized** to each user, by updating their preferences in response to user feedback
• **Evaluated** on real blog data (see paper!)
This week

We’ve looked at three features to handle the properties unique to online advertising

1. We need to handle **budgets** at the level of users and content (Matching problems)
2. We need algorithms that can operate **online** (i.e., as users arrive one-at-a-time) (AdSense)
3. We need to algorithms that exhibit an explore-exploit tradeoff (Bandit algorithms)
Further reading:

- Turning down the noise in the blogosphere (by Khalid El-Arini, Gaurav Veda, Dafna Shahaf, Carlos Guestrin)
  
Assignment 1