INSTRUCTIONS

Upload a **single file** to Gradescope for each group. All group members’ names and PIDs should be on **each** page of the submission.

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

**READING Sipser Chapter 1.3-1.4**

**KEY CONCEPTS** Regular Expressions, Regular and Non-regular languages, Pumping Lemma
1. **(10 points)** Let $\Sigma = \{a, b\}$. For each regular expression, describe the language of the regular expression by choosing one of the sets named below, or saying none if the language of the regular expression is not any of the given sets. Sets may be used more than once or not at all.

Sets:

- $A = \{ w \in \Sigma^* \mid w$ doesn’t contain the substring $bb \}$
- $B = \{ w \in \Sigma^* \mid w$ contains the substring $bb \}$
- $C = \{ w \in \Sigma^* \mid w$ starts or ends with $bb \}$
- $D = \{ w \in \Sigma^* \mid w$ starts and ends with $bb \}$
- $E = \{ w \in \Sigma^* \mid w$ does not end with $bb \}$
- $F = \{ w \in \Sigma^* \}$

Regular Expressions:

1. $bb(a \cup b)^*bb$
2. $(ba \cup a)^* \cup (ab)^*$
3. $(ba^* \cup a)^*$
4. $bb(a \cup b)^* \cup (a \cup b)^*bb$
5. $bb(a \cup b)^*bb \cup bb \cup bbb$
6. $b^*b^*(a \cup b)^* \cup (a \cup b)^*b^*b^*$
7. $(bb)^*$
8. $(a \cup ab)^* \cup (ba)^*$
9. $(a \cup b)^*(a \cup ab) \cup \varepsilon \cup b$
10. $(a^* \cup b^*)^*bb(a \cup b)^*$

2. **(10 points)** A common misconception about regular languages is that if a language is regular, then its subsets or supersets must be too. In this question, you will show that this is false. Let $\Sigma = \{0, 1\}$.

(a) Give an example of a regular language $X$ that is a subset of all nonregular languages over $\Sigma$. Briefly justify your answer. (3 points)

(b) Give an example of a regular language $A$ and a nonregular language $B$ such that $A \subseteq B$. For this part, you must choose $A$ and $B$ that are neither equal to $\emptyset$ nor to $\Sigma^*$. (3 points)

(c) Give an example of a nonregular language $C$ and a regular language $D$ such that $C \subseteq D$. For this part, you must choose $C$ and $D$ that are neither equal to $\emptyset$ nor to $\Sigma^*$. (4 points)

For each part of this problem, justify any claims you make about certain sets being regular or nonregular either by proving the claim from definitions or citing a fact proved in class/textbook.
3. (10 points) Consider the language \( L \) of odd length strings over the alphabet \( \{a, b, c\} \) such that the middle symbol is \( b \). For example, \( abcba \), \( b \), \( aaabca \) are in \( L \) but \( bb \), \( acb \), \( a \) are not in \( L \). Fill in the missing parts of the following proof to show that \( L \) is not regular.

Assume (towards a contradiction) that \( L \) is regular. Then the Pumping Lemma applies to \( L \). Let \( p \) be the pumping length of \( L \). Choose \( s \) to be the string \( \underline{\ldots} \). Since this string is in \( L \) and has length greater than or equal to \( p \), the Pumping Lemma guarantees that \( s \) can be divided into parts \( s = xyz \) such that for any \( i \geq 0 \), \( xy^iz \) is in \( L \), and that \( |y| > 0 \) and \( |xy| \leq p \). But if we let \( i = \underline{\ldots} \), we get a string \( xy^iz \) that is not in \( L \) because \( \underline{\ldots} \).

This is a contradiction. Therefore the assumption is false, and \( L \) is not regular.

4. (10 points) Consider the language \( L = \{tu \mid t \text{ and } u \text{ are strings over } \{0, 1\} \text{ with the same number of } 1\text{'s}\} \). Explain and correct the error below:

Proof that \( L \) is not regular using the Pumping Lemma:
Assume (towards a contradiction) that \( L \) is regular. Then the Pumping Lemma applies to \( L \). Let \( p \) be the pumping length of \( L \). Choose \( s \) to be the string \( 1^p0^p1^p0^p \), which is in \( L \) because \( t = 1^p0^p \) and \( u = 1^p0^p \) each have \( p \) 1’s. Since this string is in \( L \) and has length greater than or equal to \( p \), the Pumping Lemma guarantees that \( s \) can be divided into parts \( s = xyz \) such that for any \( i \geq 0 \), \( xy^iz \) is in \( L \), and that \( |y| > 0 \) and \( |xy| \leq p \). Since the first \( p \) characters of \( s \) are all 1’s and we have \( |y| > 0 \) and \( |xy| \leq p \), we know that \( y \) must be nonempty and made up of all 1’s. But if we let \( i = 2 \), we get a string \( xy^2z \) that is not in \( L \) because repeating \( y \) twice adds 1’s to \( t \) but not to \( u \), and strings in \( L \) are required to have the same number of 1’s in \( t \) as in \( u \). This is a contradiction. Therefore the assumption is false, and \( L \) is not regular.

5. (20 points) Prove that the following languages are not regular. The format of proof should be similar to problem 3.

1. \( L = \{wtw^R \mid w \in \{0, 1\}^*, t \in \{1\}^*, \text{ and both } w, t \text{ are nonempty}\} \). (10 points)

2. \( L = \{a^{2^n} \mid n \geq 0\} \) (10 points)