Midterm 2 Practice Problems

1. Give a CFG and PDA for the following languages:
   1. The set of strings over the alphabet \{a, b\} with more a’s than b’s
   2. The complement of the language \{a^n b^n | n \geq 0\}
   3. \{w#x | w^R is a substring of x for w, x \in \{0, 1\}^*\}
   4. \{x_1#x_2#...#x_k | k \geq 1, each x_i \in \{a, b\}^*, and for some i and j, x_i = x_j^R\}

2. Draw a state diagram and give an implementation level description of a TM that decides each of the following languages:
   1. \{0\}, \Sigma = \{0\}
   2. \{(00)^n\}, \Sigma = \{0\}
   3. \{0^n | n \geq 0\}, \Sigma = \{0, 1\}
   4. The set of strings that are palindromes over \{0, 1\}
   5. The set of strings over the alphabet \{0, 1, 2\} with more 0’s than 1’s and more 1’s than 2’s
   6. \{0^n | n \geq 0\} (hard)

3. Show that the class of decidable languages over \Sigma = \{0, 1\} is closed under:
   - \(skip(L) = \{w_1 w_3 w_5 ... w_n | w = w_1 w_2 ... w_n \in L\}\)
   Can you use a similar proof to show that the class of recognizable languages over \{0, 1\} is closed under skip? If you can, prove it, otherwise explain why not.

4. Show that the class of recognizable languages over \{0, 1\} is closed under:
   - \(prefix(L) = \{w | there is some string y where wy \in L\}\)
   Can you use a similar proof to show that the class of decidable languages over \{0, 1\} is closed under prefix? If you can, prove it, otherwise explain why not.

5. Prove that the Halting problem on the empty input is undecidable.

\(HaltEmpty = \{< M > : M \text{ halts on the empty input } \varepsilon\}.\)