1. True or false. Briefly justify your answers.
   a. Intersection of two regular languages is regular.
   b. If a language $L$ satisfies the pumping lemma, then $L$ is regular.
   c. There is a nonregular language $L$ such that every proper subset of $L$ is regular.
   d. Complement of a nonregular language could be either regular or nonregular.
   e. Union of nonregular languages is always nonregular.
   f. Intersection of a regular language and a nonregular language is nonregular.
   g. If $L$ is a regular language and $L'$ is a nonregular language that is disjoint from $L$, then $L \cup L'$ is nonregular.

2. Suppose $L$ is a nonregular language and $w \in L$ is a string in $L$. Prove that the language $L' = L \setminus \{w\}$ is nonregular as well.

3. Give a DFA for the following languages over $\Sigma = \{0, 1\}$, and write a regular expression for each language.
   a. $L = \{ w : w$ starts with a $0$ and has odd length or starts with a $1$ and has even length $\}$
   b. $L = \{ w :$ every odd position of $w$ is $1$ $\}$
   c. $L$ is all strings $w$ such that in every prefix of $w$, the number of $0$’s and $1$’s differ by at most one.
4. Give an NFA for the following languages over $\Sigma = \{0, 1\}$, and write a regular expression for each language.
   a. $L = \{xy : x has at most 5 symbols and every odd position of y is a 1\}$
   b. $L = \{w : w has an even number of 0's or has exactly three 1's\}$

5. Construct equivalent DFAs for the following NFAs. Show your work.
   a.
   ![Diagram]
   b.
   ![Diagram]

6. Prove the following languages are nonregular.
   a. $L = \{0^n1^m0^n : n, m \geq 0 \}$
   b. $L = \{w : w is a palindrome \}$
   c. $L = \{wtw : w, t are arbitrary strings of length at least one \}$
   d. $L = \{0^m1^n : m \geq n \}$
   e. $L = \{a^ib^jc^k : i, j, k \geq 0 \text{ and if } i = 0, \text{ then } j = k \}$
   f. $L = \{a^{2n}b^{3n}a^n : n \geq 0 \}$

7. Prove that the union of two regular languages is regular.