Instructions

Upload a single file to Gradescope for each group. All group members’ names and PIDs should be on each page of the submission.

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Reading Sipser Sections 3.1, 3.2, 3.3, 3.4, 4.1

Key Concepts Turing machines, recognizable languages, decidable languages, equivalent computational models.

1. (10 points) Let $L = \{ w \in \{0,1\}^* : |w| = 3^n, n \geq 0 \}$.
   
   (a) Give a high-level description of a Turing machine that decides this language.
   
   (b) Draw the state diagram of the Turing machine.

2. (10 points) Prove the set of languages recognized by a Turing machine is closed under intersection by giving a high level description of a Turing machine that recognizes the intersection of their two languages.

3. (10 points) Prove if a language $L$ is recognizable, then so is $L^*$. Again, do this with a high-level description. (Hint: first solve for $L \circ L$ and think in terms of the number of steps in the Turing machines.)

4. (10 points) Define a de-numerator of a language $L$ as a machine that prints all the strings not in $L$ in an order of non-decreasing length. Prove a language is Turing decidable if it has a de-numerator. (Hint: consider the two disjoint cases, first when the complement of $L$ is finite and second when it is infinite. In the first case you can show $L$ is decidable without the de-numerator.)

5. (10 points) Let the language $A$ be the pairs $\langle D, w \rangle$ where $D$ is (an encoding of) a DFA with alphabet $\Sigma$ and $w \in \Sigma^*$ is a string in the language of $D$. Formally, $A = \{ \langle D, w \rangle : D$ is a DFA and $w \in L(D) \}$. Prove this language is decidable.