Announcements

- Homework 1 is due today, 11:59 PM
  - Submit answers and results via Gradescope
  - Submit code via email
- Homework 2 will be assigned today
- Reading:
  - Chapter 7: Stereopsis

Binocular Stereopsis: Mars
Given two images of a scene where relative locations of cameras are known, estimate depth of all common scene points.

Two images of Mars (Viking Lander)

An Application: Mobile Robot Navigation

The Stanford Cart,

The INRIA Mobile Robot, 1990.

Commercial Stereo Heads

Trinocular stereo

Binocular stereo
Stereo can work well

Need for correspondence

Triangulation

Stereo Vision Outline

- Offline: Calibrate cameras & determine
  - "epipolar geometry"
- Online
  1. Acquire stereo images
  2. Rectify images to convenient epipolar geometry
  3. Establish correspondence
  4. Estimate depth

Stereo Vision Outline

- Offline: Calibrate cameras & determine
  - “epipolar geometry”
- Online
  1. Acquire stereo images
  2. Rectify images to convenient epipolar geometry
  3. Establish correspondence
  4. Estimate depth
Two Approaches

1. Feature-Based
   - From each image, process “monocular” image to obtain cues (e.g., corners, lines).
   - Establish correspondence between the two images.

2. Area-Based
   - Directly compare image regions between the two images.

Human Stereopsis: Binocular Fusion

How are the correspondences established?

Julesz (1971): Is the mechanism for binocular fusion a monocular process or a binocular one?
• There is anecdotal evidence for the latter (camouflage).

Random dot stereograms provide an objective answer.

Random Dot Stereograms
Was Rembrandt Stereo Blind?
- Detail of a 1639 etching.

- In Rembrandt's painted self-portraits (left panel) in which the eyes are clearly visible, his left eye frequently looks straight out and the right off to the side. It is the opposite in his etchings (right panel).

Need for correspondence

Where does a point in the left image match in the right image?
Epipolar Constraint
• Potential matches for \( p \) have to lie on the corresponding epipolar line \( l' \).
• Potential matches for \( p' \) have to lie on the corresponding epipolar line \( l \).

Epipolar Geometry
• Epipolar Plane
• Epipoles
• Epipolar Lines

Family of epipolar Planes
Family of planes \( \pi \) and lines \( l \) and \( l' \)
Intersection in \( e \) and \( e' \)

Skew Symmetric Matrix & Cross Product
• The cross product \( a \times b \) of two vectors \( a \) and \( b \) can be expressed as a matrix vector product \( [a]b \) where \( [a] \) is the skew symmetric matrix:
\[
[a] = \begin{bmatrix}
0 & -a_3 & a_2 \\
 a_3 & 0 & -a_1 \\
- a_2 & a_1 & 0
\end{bmatrix}
\]
• A matrix \( S \) is skew symmetric if and only if \( S = -S^T \)

Essential Matrix
(Lonquid-Higgins, 1981)

Properties of the Essential Matrix
\( p'Ep' = 0 \) with \( E = [t, \|R\] \)
• \( E'p' \) is the epipolar line associated with \( p' \).
• \( E'Tp \) is the epipolar line associated with \( p \).
• \( E'e=0 \) and \( E'e=0 \).
• \( E \) is singular (rank 2).
• \( E \) has two equal non-zero singular values (Huang and Faugeras, 1989).
Calibration

Determine intrinsic parameters and extrinsic relation of two cameras

The Eight-Point Algorithm (Longuet-Higgins, 1981)

\[
\begin{pmatrix}
F_{11} & F_{12} & F_{13} \\
F_{21} & F_{22} & F_{23} \\
F_{31} & F_{32} & F_{33}
\end{pmatrix}
\begin{pmatrix}
w'1 \\
w'2 \\
w'3
\end{pmatrix} = 0
\]

Consider 8 points \((u_i, v_i), (u'_i, v'_i)\)

Set \(F_{33}\) to 1

Solve for \(F_{11}\) to \(F_{32}\)

For more than 8 points, solve using linear least squares

Alternatively, view this as system of homogenous equations in \(F_{11}\) to \(F_{33}\)

Solve as Eigenvector corresponding to the smallest Eigenvalue of matrix created from the image data.

Equivalent to solving

\[
\sum_{i=1}^{n} \left( p_i^T F p'_i \right)^2
\]

under the constraint

\(F^T F = I\).

Epipolar geometry example

The epipolar constraint is given by:

\[ (Ap)^T (A'q') = q^T (A'EA') q = q^T F q' = 0 \]

where \(p\) and \(p'\) are called homogeneous normalized image coordinates of points in the two images.

Without calibration, we can still identify corresponding points in two images, but we can’t convert to 3-D coordinates. However, the relationship between the calibrated coordinates \(p\) and uncalibrated coordinates \(q\) can be expressed as \(p = Aq\) and \(p' = A'q'\)

Therefore, we can express the epipolar constraint as:

\[ (Aq)^T (A'q') = q^T (A'EA') q = q^T F q' = 0 \]

Two-View Geometry

Essential Matrix \(E\)

- Rank 2
- Calibrated
- Normalized coordinates
- 5 degrees of freedom
  - Camera rotation
  - Direction of camera translation
- Similarity reconstruction

Fundamental Matrix \(F\)

- Rank 2
- Uncalibrated
- Image coordinates
- 7 degrees of freedom
  - Homogeneous matrix to scale
  - \(\det F \neq 0\)
- Projective reconstruction
Example: converging cameras
courtesy of Andrew Zisserman

Example: motion parallel with image plane
(simple for stereo $\rightarrow$ rectification)
courtesy of Andrew Zisserman

Example: forward motion
courtesy of Andrew Zisserman

Next Lecture
- Early vision: multiple images
  - Stereo
- Reading:
  - Chapter 7: Stereopsis