Photometric Stereo

Computer Vision I
CSE 252A
Lecture 4

Announcements
• Homework 0 is due today by 11:59 PM
• Homework 1 will be assigned today
  – Due Wed, Oct 19, 11:59 PM
• Reading:
  – Section 2.2.4: Photometric Stereo
    • Shape from Multiple Shaded Images

Shading reveals 3-D surface geometry

Two shape-from-X methods that use shading
• Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.

• Photometric stereo: Single viewpoint, multiple images under different lighting.

Photometric Stereo Rigs:
One viewpoint, changing lighting

An example of photometric stereo
surface (albedo textured mapped on surface)
albedo (surface normals)
Photometric stereo

- Single viewpoint, multiple images under different lighting.
  1. Arbitrary known BRDF, known lighting
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting.

I. Photometric Stereo: General BRDF and Reflectance Map

BRDF

- Bi-directional Reflectance Distribution Function
  \[ \rho(\theta_{in}, \phi_{in} ; \theta_{out}, \phi_{out}) \]
- Function of
  - Incoming light direction: \( \theta_{in}, \phi_{in} \)
  - Outgoing light direction: \( \theta_{out}, \phi_{out} \)
- Ratio of incident irradiance to emitted radiance

Coordinate system

Gradient Space (p,q)

Image Formation

For a given point A on the surface, the image irradiance \( E(x,y) \) is a function of

1. The BRDF at \( A \)
2. The surface normal at \( A \)
3. The direction of the light source
Reflectance Map

Let the BRDF be the same at all points on the surface, and let the light direction $s$ be a constant.
1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have $E(p,q)$.

Example Reflectance Map: Lambertian surface

For lighting from front

Example Reflectance Map:
Lambertian surface

What does the intensity (Irradiance) of one pixel in one image tell us?
It constrains the surface normal projecting to that point to a curve

Reflectance Map of Lambertian Surface

Two Light Sources
Two reflectance maps

A third image would disambiguate match

Three Source Photometric stereo:
Step 1

Offline:
Using source directions & BRDF, construct reflectance map for each light source direction. $R_1(p,q)$, $R_2(p,q)$, $R_3(p,q)$

Online:
1. Acquire three images with known light source directions. $E_1(x,y)$, $E_2(x,y)$, $E_3(x,y)$
2. For each pixel location $(x,y)$, find $(p,q)$ as the intersection of the three curves
   $R_1(p,q)=E_1(x,y)$
   $R_2(p,q)=E_2(x,y)$
   $R_3(p,q)=E_3(x,y)$
3. This is the surface normal at pixel $(x,y)$. Over image, the normal field is estimated
Normal Field

Plastic Baby Doll: Normal Field

Next step:
Go from normal field to surface

Recovering the surface \( f(x,y) \)

Many methods: Simplest approach
1. From estimate \( \mathbf{n} = (n_x, n_y, n_z) \), \( p = n_x/n_z \), \( q = n_y/n_z \)
2. Integrate \( p = df/dx \) along a row \((x,0)\) to get \( f(x,0) \)
3. Then integrate \( q = df/dy \) along each column starting with value of the first row

Integrability. If \( f(x,y) \) is the height function, we expect that
\[
\frac{\partial f}{\partial y} \frac{\partial}{\partial x} = \frac{\partial}{\partial \bar{y}} \frac{\partial f}{\partial \bar{x}}
\]

In terms of estimated gradient space \((p,q)\), this means:
\[
\hat{p} = \frac{\partial q}{\partial \bar{x}} \quad \hat{q} = \frac{\partial p}{\partial \bar{y}}
\]

But since \( p \) and \( q \) were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold

What might go wrong?

• Height \( z(x,y) \) is obtained by integration along a curve from \((x_0, y_0)\):
  \[
  z(x,y) = z(x_0,y_0) + \int_{(x_0,y_0)}^{(x,y)} (pdx + qdy)
  \]
  • If one integrates the derivative field along any closed curve, one expects to get back to the starting value.
  • Might not happen because of noisy estimates of \((p,q)\)
Horn’s Method

*Robot Vision, B.K.P. Horn, 1986*

- Formulate estimation of surface height $z(x,y)$ from gradient field by minimizing cost functional:
  \[
  \int_{\text{Image}} (z_x - p)^2 + (z_y - q)^2 \, dx \, dy
  \]
  where $(p,q)$ are estimated components of the gradient while $z_x$ and $z_y$ are partial derivatives of best fit surface.
- Solved using calculus of variations – iterative updating
- $z(x,y)$ can be discrete or represented in terms of basis functions.
- Integrability is naturally satisfied.

II. Photometric Stereo:
Lambertian Surface, Known Lighting

Lambertian Surface

At image location $(u,v)$, the intensity of a pixel $x(u,v)$ is:

\[
e(u,v) = [a(u,v) \hat{n}(u,v)] \cdot [s_0 \ s] = b(u,v) \cdot s
\]

where
- $a(u,v)$ is the albedo of the surface projecting to $(u,v)$.
- $\hat{n}(u,v)$ is the direction of the surface normal.
- $s_0$ is the light source intensity.
- $s$ is the direction to the light source.

If the light sources $s_1$, $s_2$, and $s_3$ are known, then we can recover $b$ from as few as three images. (Photometric Stereo: Silver 80, Woodham 81).

\[
[e_1 \ e_2 \ e_3] = b^T[s_1 \ s_2 \ s_3]
\]

- i.e., we measure $e_1$, $e_2$, and $e_3$ and we know $s_1$, $s_2$, and $s_3$. We can then solve for $b$ by solving a linear system.
- Normal $\hat{n} = b/|b|$ and albedo $a = |b|$

What if we have more than 3 Images?
Linear Least Squares

\[
[e_1 \ e_2 \ ... \ e_n] = \hat{b}^T[s_1 \ s_2 \ ... \ s_n]
\]

Let the residual be

\[
r = e - \hat{b}^T S b
\]

Squaring this:

\[
r^T r = (e - \hat{b}^T S b)^T (e - \hat{b}^T S b) = e^T e - 2b^T S^T e + b^T S^T S b
\]

Zero derivative is a necessary condition for a minimum, or

\[
2S^T e - 2S^T S b = 0; \quad -2S^T e + 2S^T S b = 0;
\]

Solving for $b$ gives

\[
b = (S^T S)^{-1} S^T e
\]
Recovered albedo

Recovered normal field

Surface recovered by integration

An example of photometric stereo

Next Lecture

- Illumination cones
  - III. Photometric Stereo with unknown lighting and Lambertian surfaces
- Reading:
  - What Is the Set of Images of an Object under All Possible Illumination Conditions?