Announcements

• Course website
  – http://cseweb.ucsd.edu/classes/fa16/cse252A-a/

• Piazza

• Homework 0 will be assigned today
  – MATLAB and LaTeX
  – Due Wed, Oct 5, 11:59 PM

• Wait list

• Reading:
  – Chapters 1: Geometric camera models

Earliest Surviving Photograph

• First photograph on record, “la table service” by
  Nicephore Niepce in 1822.
• Note: First photograph by Niepce was in 1816.

How Cameras Produce Images

• Basic process:
  – photons hit a detector
  – the detector becomes charged
  – the charge is read out as brightness

• Sensor types:
  – CCD (charge-coupled device)
    • high sensitivity
    • high power
    • cannot be individually addressed
    • blooming
  – CMOS
    • simple to fabricate (cheap)
    • lower sensitivity, lower power
    • can be individually addressed

Images are two-dimensional patterns of brightness values.

Effect of Lighting: Monet
Change of Viewpoint: Monet

Haystack at Chailly at sunrise (1865)

Image Formation: Outline

- Geometric camera models
- Light and shading
- Color

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

Camera Obscura

- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer)

Camera Obscura

Jetty at Margate England, 1898.

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)
Distant objects are smaller

Geometric properties of projection
- 3-D points map to points
- 3-D lines map to lines
- Planes map to whole image or half-plane
- Polygons map to polygons

Important point to note: Angles & distances not preserved, nor are inequalities of angles & distances.
- Degenerate cases:
  - line through focal point project to point
  - plane through focal point projects to a line

In the perspective image, two parallel lines meet at a point

Parallel lines meet in the image
- Formed by line through O
- Parallel to the given line(s)
- A single line can have a vanishing point

Projective geometry provides an elegant means for handling these different situations in a unified way, and homogenous coordinates are a way to represent entities (points & lines) in projective spaces.

Vanishing points
Different directions correspond to different vanishing points
Vanishing Points

Beyond the pinhole Camera
Getting more light – Bigger Aperture

Pinhole Camera Images with Variable Aperture

The reason for lenses
We need light, but big pinholes cause blur.

Thin Lens

Thin Lens: Center

• Rotationally symmetric about optical axis.
• Spherical interfaces.
• All rays that enter lens along line pointing at O emerge in same direction.
**Thin Lens: Focus**

Parallel lines pass through the focus, \( F \)

**Thin Lens: Image of Point**

- All rays passing through lens and starting at \( P \) converge upon \( P' \)
- So light gather capability of lens is given the area of the lens and all the rays focus on \( P' \) instead of become blurred like a pinhole

**Thin Lens: Image of Point**

\[
\frac{1}{z'} = \frac{1}{z} = \frac{1}{f}
\]

Relation between depth of Point (-\( Z \)) and the depth where it focuses (\( Z' \))

**Thin Lens: Image Plane**

A price: Whereas the image of \( P \) is in focus, the image of \( Q \) isn’t.

**Thin Lens: Aperture**

- Smaller Aperture -> Less Blur
- Pinhole -> No Blur

**Equation of Perspective Projection**

Cartesian coordinates:
- We have, by similar triangles, that \((x', y', z') = (f' x/z, f' y/z, f')\)
- Establishing an image plane coordinate system at \( C' \) aligned with \( i \) and \( j \), image coordinates of the projection of \( P \) are \((x, y, z) \rightarrow (f' x/z, f' y/z, f')\)
The equation of projection

Cartesian coordinates:
\((X,Y,Z)\rightarrow(f \frac{x}{Z}, f \frac{y}{Z}) = (x', y')\)

Homogenous Coordinates and Camera matrix:
\[
\begin{bmatrix}
X \\
Y \\
Z \\
w
\end{bmatrix} =
\begin{bmatrix}
f & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
w
\end{bmatrix}
\]

What if camera coordinate system differs from world coordinate system?

Euclidean Coordinate Systems

Coordinate Change: Translation Only

\(X' = X + t\)

Coordinate Change: Rotation Only

\(X' = RX\)

Coordinate Changes: Rotation and Translation

\(X' = RX + t\)
Some points about SO(n)

- \( SO(n) = \{ R \in \mathbb{R}^{n \times n} : R^T R = I, \det(R) = 1 \} \)
  - \( SO(2) \): rotation matrices in plane \( \mathbb{R}^2 \)
  - \( SO(3) \): rotation matrices in 3-space \( \mathbb{R}^3 \)
- Forms a Group under matrix product operation:
  - Identity
  - Inverse
  - Associative
  - Closure
- Closed (finite intersection of closed sets)
- Bounded \( R_{ij} \in [-1, +1] \)
- Does not form a vector space.
- Manifold of dimension \( n(n-1)/2 \)
  - \( \text{Dim}(SO(2)) = 1 \)
  - \( \text{Dim}(SO(3)) = 3 \)

Parameterizations of SO(3)

- Even though a rotation matrix is 3x3 with nine numbers, it only has three degrees of freedom. It can be parameterized with three numbers. There are many parameterizations.
- Other common parameterizations
  - Euler Angles
  - Axis Angle
  - Quaternions
  - four parameters; homogeneous

Rotation: Homogenous Coordinates

- About z axis

\[
\begin{pmatrix}
  x' \\
  y' \\
  z' \\
  1
\end{pmatrix} =
\begin{pmatrix}
  \cos \theta & -\sin \theta & 0 & 0 \\
  \sin \theta & \cos \theta & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  x \\
  y \\
  z \\
  1
\end{pmatrix}
\]

- About x axis:

\[
\begin{pmatrix}
  x' \\
  y' \\
  z' \\
  1
\end{pmatrix} =
\begin{pmatrix}
  1 & 0 & 0 & 0 \\
  0 & \cos \theta & -\sin \theta & 0 \\
  0 & \sin \theta & \cos \theta & 0 \\
  0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  x \\
  y \\
  z \\
  1
\end{pmatrix}
\]

- About y axis:

\[
\begin{pmatrix}
  x' \\
  y' \\
  z' \\
  1
\end{pmatrix} =
\begin{pmatrix}
  \cos \theta & 0 & \sin \theta & 0 \\
  0 & 1 & 0 & 0 \\
  -\sin \theta & 0 & \cos \theta & 0 \\
  0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  x \\
  y \\
  z \\
  1
\end{pmatrix}
\]

Euler Angles: Roll-Pitch-Yaw

- Composition of rotations

\[
R = R_\gamma(\gamma) \cdot R_\beta(\beta) \cdot R_\alpha(\alpha)
= \begin{pmatrix}
  \cos \gamma & -\sin \gamma & 0 & 1 \\
  \sin \gamma & \cos \gamma & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
  \cos \beta & 0 & \sin \beta \\
  0 & 1 & 0 \\
  -\sin \beta & 0 & \cos \beta \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  \cos \alpha & 0 & \sin \alpha \\
  0 & 1 & 0 \\
  -\sin \alpha & 0 & \cos \alpha \\
  0 & 0 & 1
\end{pmatrix}
\]

What if camera coordinate system differs from world coordinate system?

\[
X_{\text{Camera}} = R \cdot X_{\text{World}} + t
\]
**Intrinsic parameters**

- 3x3 homogenous matrix
- Focal length
- Principal Point
- Units (e.g. pixels)
- Pixel Aspect ratio

**Camera Calibration**

Given \( n \) points \( P_1, \ldots, P_n \) with known positions and their images \( p_1, \ldots, p_n \), estimate intrinsic and extrinsic camera parameters

- See Text book for how to do it.
- Camera Calibration Toolbox for Matlab (Bouguet)
  - http://www.vision.caltech.edu/bouguetj/calib_doc/

**Camera parameters**

- **Extrinsic Parameters**: Since camera may not be at the origin, there is a rigid transformation between the world coordinates and the camera coordinates.
- **Intrinsic parameters**: Since scene units (e.g., cm) differ image units (e.g., pixels) and coordinate system may not be centered in image, we capture that with a 3x3 transformation comprised of focal length, principal point, pixel aspect ratio, and skew.

\[
\begin{pmatrix}
  x \\
  y \\
  w
\end{pmatrix} = \text{Transformation} \begin{pmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1
\end{pmatrix} \text{Rigid Transformation} \begin{pmatrix}
  X \\
  Y \\
  Z
\end{pmatrix}
\]

\[
\begin{pmatrix}
  x \\
  y \\
  w
\end{pmatrix} = \text{intrinsic parameters} \begin{pmatrix}
  0 & 0 & 1 \\
  0 & 1 & 0
\end{pmatrix} \text{extrinsic parameters} \begin{pmatrix}
  X \\
  Y \\
  Z
\end{pmatrix}
\]

**Camera Models**

- Perspective Projection
- Affine Camera Model
- Scaled Orthographic Projection
- Parallel Projection

**Other camera models**

- Generalized camera – maps points lying on rays and maps them to points on the image plane.

- Omnicam (hemispherical)
- Light Probe (spherical)
Some Alternative “Cameras”

Next Lecture

• Image Formation: Light and Shading
• Reading:
  – Chapter 2: Light and Shading