INSTRUCTIONS

Homework should be done in groups of **one to three** people. You are free to change group members at any time throughout the quarter. Problems should be solved together, not divided up between partners. A **single representative** of your group should submit your work through Gradescope. Submissions must be received by **10:00pm** on the due date, and there are no exceptions to this rule.

You will be able to look at your scanned work before submitting it. Please ensure that your submission is **legible** (neatly written and not too faint) or your homework may not be graded.

Students should consult their textbook, class notes, lecture slides, instructors, TAs, and tutors when they need help with homework. Students should not look for answers to homework problems in other texts or sources, including the internet. You may ask questions about the homework in office hours, but **not on Piazza**.

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should **always explain** how you arrived at your conclusions and **justify your answers** with mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to **convince the reader** that your results and methods are sound.

For questions that require pseudocode, you can follow the same format as the textbook, or you can write pseudocode in your own style, as long as you specify what your notation means. For example, are you using “=” to mean assignment or to check equality? You are welcome to use any algorithm from class as a subroutine in your pseudocode. For example, if you want to sort list A using InsertionSort, you can call InsertionSort(A) instead of writing out the pseudocode for InsertionSort.

** REQUIRED READING ** Rosen 10.1, 10.2, 10.3, 10.4 through Theorem 1, 10.5 through Example 7.

** KEY CONCEPTS** Graphs (definitions, modeling problems using graphs), Hamiltonian tours, Eulerian tours, Fleury’s algorithm.
1. Before you watch TV at night, you look through the schedule of TV shows which includes all different programs on TV that night, the times they will be on TV, and the channel on which they will be showing. You compile a list of shows you are interested in watching, with program \( i \) beginning at time \( b_i \), ending at time \( e_i \), and showing on channel \( c_i \). Assume that it does not take you any time to change channels, that you will not watch any programs unless you are interested in them, and that you will only watch programs in full.

(a) (4 points) Describe how to model this situation using a graph, in such a way that paths in your graph represent possible sequences of interesting TV programs you can watch that night. Be sure to include what the vertices represent, whether your edges are directed or undirected, and a rule you can use to determine when two vertices are connected with an edge.

(b) (2 points) What problem would you need to solve in your graph to determine the maximum number of interesting TV programs you can watch that night?

(c) (2 points) Will your graph be connected? Always, sometimes, or never?

(d) (2 points) Will your graph include cycles? Always, sometimes, or never?

2. In a card game like Uno, each card has a color (red, yellow, blue, or green) and a number (0 through 9). Cards are played one at a time into a pile, where a card can only be added to the pile if it has the same color or number as the top card in the pile. For example, the pile could have a red 7, then a blue 7, then a blue 0.

Suppose you have a group of Uno cards and you want to know if you can use them all to form a pile according to the rules of the game. If so, you want to know all the different piles that are possible using all your cards. Note: we’ll consider two piles the same if they have all the same cards in the reverse order.

(a) (4 points) Given any group of Uno cards, describe how to draw a graph that will help you to determine all possible piles you can make with your entire group of cards. Carefully describe what the vertices of your graph represent, and when two vertices are connected with an edge. Specify whether your graph is directed or undirected.

(b) (2 points) Draw the graph described in part (a) for the example group of Uno cards given below. Draw your graph as a planar graph, so that no edges cross.

- blue 3
- red 2
- yellow 3
- green 9
- blue 5
- red 0
- green 2
- blue 8
- red 3

(c) (2 points) For the example group of Uno cards given in part (b), use your graph to help you list all possible piles that can be formed, or say why it is impossible to form a pile according to the rules of the game.
3. A domino is a tile game piece that has two halves, and each half has a number of dots on it, anywhere from 0 to 6 dots per half. To form a chain, you must line up your dominoes so that adjacent halves from different dominoes have the same number of dots. For example, the following three dominoes are arranged in a chain.

![Dominoes Chain Image]

Suppose you have a group of dominoes and you want to know if you can use them all to form a chain. If so, you want to know all the different domino chains that are possible using all your dominoes. Note: we’ll consider two chains the same if they have all the same dominoes in the reverse order.

(a) (4 points) Given any group of dominoes, describe how to draw a graph that will help you to determine all possible domino chains you can make with your entire group of dominoes. Carefully describe what the vertices of your graph represent, and when two vertices are connected with an edge. Specify whether your graph is directed or undirected.

(b) (2 points) Draw the graph described in part (a) for the example group of dominoes given below. Draw your graph as a planar graph, so that no edges cross. Each domino is named by the number of dots appearing on its two halves. Note that dominoes are tiles that can be rotated, so there is no difference between the domino 3, 4 and the domino 4, 3.

- 3, 6
- 3, 0
- 2, 2
- 0, 1
- 4, 5
- 4, 0
- 6, 6
- 1, 2
- 3, 4
- 2, 0

(c) (2 points) For the example group of dominoes given in part (b), use your graph to help you draw all possible domino chains, or say why it is impossible to form a domino chain.

4. A d-regular graph is a simple undirected graph in which every vertex has degree d.

(a) (2 points) Draw a 3-regular graph with 5 vertices, or prove why it is impossible.

(b) (2 points) Draw a 3-regular graph with 6 vertices, or prove why it is impossible.

(c) (2 points) For which values of n > 3 is it possible to draw a 3-regular graph with n vertices?

(d) (2 points) For which values of d does a d-regular graph with 10 vertices have an Eulerian circuit?

(e) (2 points) For which values of d does a d-regular graph with 10 vertices have an Eulerian tour that starts and ends at different vertices?
5. For $n \geq 3$, define the grid graph $G_n$ to be the undirected graph with $n^2$ vertices arranged in an $n \times n$ grid. The graphs $G_3$ and $G_4$ are shown below.

(a) (3 points) What is the minimum number of colors needed to color the vertices of the graph $G_n$ so that each edge has different colored endpoints? As an example, show how you can color $G_5$ using the minimum number of colors.

(b) (2 points) Suppose $n \geq 3$ is odd. How many vertices of each color will there be in the coloring you described in part (a)?

(c) (6 points) Use your answer to part (b) to prove that when $n \geq 3$ is odd, the graph $G_n$ has no Hamiltonian tour starting at a vertex adjacent to a corner vertex.

(d) (3 points) When $n \geq 3$ is even, the graph $G_n$ does have a Hamiltonian tour starting at a vertex adjacent to a corner vertex. Draw such a Hamiltonian tour on each of the graphs $G_4$, $G_6$, and $G_8$. 