1. Consider a TM variant with a single two-dimensional tape, where the tape head can move either one step to the left, right, up or down. The input is arranged in a $\sqrt{n} \times \sqrt{n}$ grid, with the first $\sqrt{n}$ symbols on the first row, the second $\sqrt{n}$ symbols on the second row, and so on (there may be some empty spaces in the last row if $n$ isn’t a perfect square.)

Show that this model can simulate a $k$-tape TM running in time $T(n)$ in time $O(T(n)^{3/2})$. Then show that there is a language solvable in linear time on a two-tape machine that requires $\Omega(n^{3/2})$ time with a single two dimensional tape.

2. It would be nice to have a programming language PL where: A) (Recognizability) we could computably tell whether a string was a valid PL program; B) (Termination guarantee) given a valid PL program and an input, we could simulate the program on the input computably, thus guaranteeing that all programs in our language halt; and C) (Generality) for every recursive language $L$, there is a valid PL program that computes membership in $L$.

Show that no programming language exists having all three properties.

3. Consider two kinds of constructions, making a new language from an existing language, $L$: $One\text{-}of\text{-}TwoL = \{(x_1, x_2) : \text{exactly one of } x_1 \in L \text{ and } x_2 \in L\}$. $Maj_3L = \{(x_1, x_2, x_3) : \text{at least two of } x_1, x_2, x_3 \in L\}$.

We say that a class is closed under a construction if whenever $L$ is in the class, the constructed language is in the class as well. For each of the constructions above, is $COMP$ closed under the construction? Is $CE$ closed under the construction, where $CE$ is the class of computably enumerable languages (also called $RE$)? Prove all four parts of your answer (but some will be shorter than others).