Problem 1. Let $P(x) = \text{“} x \text{ is prime”}$ and $Q(x) = \text{“} x \text{ is divisible by 4”}$ defined over the universe of integer numbers. Express the following predicates using quantifiers $P, Q$. You are also allowed to use addition and/or multiplication if needed.

(a) There exists an integer which is divisible by 4

(b) If an integer is divisible by 4, it is not prime

(c) The product of two prime numbers is not prime

Problem 2. For each of the following sentences, answer the following questions:

(i) Translate the sentence into logic form using the “predicates” defined below

(ii) Negate the translated sentence.

If there are any ambiguities of these sentence in English, state your interpretation.

(a) Some courses require some prerequisite but don’t require all courses as prerequisites.
Predicate: $\text{PREREQ}(a,b)$ means $a$ requires $b$ as a prerequisite.

(b) You can satisfy some people all of the time, and all of the people some of the time, but you cannot satisfy all of the people all of the time.
Predicate: $\text{CANSATISFY}(p,t)$ means you can satisfy person $p$ at time $t$. 

**Problem 3.** For each of the following statements, decide if it is a CNF, a DNF, neither or both.

(a) \( \neg p \land q \)

(b) \( \neg (p \lor q) \)

(c) \( \neg (p \lor \neg q) \lor \neg r \)

(d) \( (p \lor \neg r) \land (p \land q) \land (\neg p \land q \lor \neg r) \)

(e) \( (\neg p \lor q) \land ((\neg p \lor q) \land (s \land \neg r)) \)

**Problem 4.** For each of the following Boolean expressions:

(i) \( \neg (q \rightarrow \neg p) \rightarrow (\neg p \lor \neg q) \)

(ii) \( (q \rightarrow p) \rightarrow (p \rightarrow q) \)

Answer the following questions:

(a) Recall from class that each predicate can be written as both a CNF and a DNF. Decide which one is more efficient (requires less connectives) for this Boolean expression and rewrite it as the more efficient form. If both CNF and DNF requires the same number of connectivity, rewrite it as either.

(b) Is this a tautology? a contradiction?  

**Problem 5.** In which domain are the following statements true? (i) integers (ii) rationals (iii) real numbers (iv) prime numbers (v) \{0,1,2\}.

For each statement, choose all that apply:

(a) \( \forall z \forall x \forall y \ (x \neq y \land y \neq z \land x \neq z) \rightarrow (z \neq x \times y) \)
(b) \( \forall z \ z \neq 0 \rightarrow \forall x \exists y \ z = \frac{x}{y} \)

**Problem 6.** Use inference rules to show that the following Boolean expression is always true (a tautology).

\[
[(p \land \lnot(p \lor q)) \lor (p \land q)] \rightarrow p
\]