Image pyramids

Residual values are near zero

Lowest resolution is approximation image, not residual image

1D discrete convolution using a digital filter

Input: unit impulse

Output: filter coefficients

1D subband coding, two bands

Input sequence

Magnitude of Fourier transform for filters in analysis filter bank

Approximation

Decoding

Vertical detail

Horizontal detail

Diagonal detail

Coding

Separable filters

Low band

High band

Low band

High band

Half rows of input image

Diagonal detail

Quarter size (and number of pixels) of input image

2D subband coding, four bands

Low band

High band

Half rows and columns of input image

Low band

High band

Half rows of input image

Coding

Approximation

Vertical detail

Horizontal detail
2D subband coding, four bands

Approximation

Horizontal detail

Vertical detail

Diagonal detail

Scaling function spaces

Figure 7.12
The nested function spaces spanned by a scaling function.

Figure 7.13
The relationship between scaling and wavelet function spaces.

Scaling and wavelet function spaces

Haar scaling functions

$\psi_{0,0}(x) = \phi(x)$

$f(x)$ in $V_1$

$\phi_{0,0}(x)$ in $V_1$

$\phi_{0,1}(x)$

Haar wavelet functions

$\psi_{1,0}(x)$

$f(x)$ in $V_1$

$f(x)$ in $V_2$

$\psi_{2,0}(x)$

approximation

$\psi_{2,1}(x)$

detail

Wavelet series expansion using Haar wavelets

A wavelet series expansion of $f(x) = \phi$ using Haar wavelets.
1D continuous wavelet transform

2D discrete wavelet transform

Haar transform $H_2$ as analysis filters

Wavelet-based edge detection
Wavelet-based noise removal

Noisy image

Zero highest resolution details

Zero details for all levels

Threshold details

Data redundancy in images

Coding redundancy

Spatial redundancy

Irrelevant information

Does not need all 8 bits

Information is unnecessarily replicated

Information is not useful

Fidelity criteria subjective (qualitative)

<table>
<thead>
<tr>
<th>Value</th>
<th>Rating</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Excellent</td>
<td>An image of extremely high quality, as good as you could desire.</td>
</tr>
<tr>
<td>2</td>
<td>Fine</td>
<td>An image of high quality, providing enjoyable viewing. Interference is not objectionable.</td>
</tr>
<tr>
<td>3</td>
<td>Passable</td>
<td>An image of acceptable quality. Interference is not objectionable.</td>
</tr>
<tr>
<td>4</td>
<td>Marginal</td>
<td>An image of poor quality you would not watch it.</td>
</tr>
<tr>
<td>5</td>
<td>Inferior</td>
<td>Always worse than others.</td>
</tr>
<tr>
<td>6</td>
<td>Unusable</td>
<td>An image so bad that you could not watch it.</td>
</tr>
</tbody>
</table>

Objective (quantitative) quality

rms error (in intensity levels)

Objective (quantitative) quality

Subjective (qualitative) quality, relative

Lower is better

(a) (b) (c)

Symbol-based coding

(0, 2) (3, 10) ...

Token Symbol Triplet

0          0 (0, 2, 10)
1          1 (3, 10, 1)
2          2 (3, 18, 2)
3          3 (3, 26, 1)
4          4 (3, 34, 7)
5          5 (3, 42, 0)
Predictive coding

Wavelet coding

<table>
<thead>
<tr>
<th>Decomposition Level</th>
<th>Approximation Coefficient Image</th>
<th>Truncated Coefficients (%)</th>
<th>Reconstruction Error (rms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>256 x 256</td>
<td>74.7%</td>
<td>3.27</td>
</tr>
<tr>
<td>2</td>
<td>128 x 128</td>
<td>95.7%</td>
<td>4.23</td>
</tr>
<tr>
<td>3</td>
<td>64 x 64</td>
<td>95.1%</td>
<td>4.64</td>
</tr>
<tr>
<td>4</td>
<td>32 x 32</td>
<td>95.0%</td>
<td>4.61</td>
</tr>
<tr>
<td>5</td>
<td>16 x 16</td>
<td>95.5%</td>
<td>4.63</td>
</tr>
</tbody>
</table>

Table 8.14: Decomposition level impact on wavelet coding; the 312 x 512 image of Eq. (59)(a).

Detail coefficients below 25 are truncated to zero.

JPEG-2000

Image watermarking system
Visible watermark

Invisible watermark

Invisible watermark

Reflection and translation

Structuring elements

Erosion
Morphological operations

- Erosion Shrinks
- Dilation Expands

Structuring element rolls along inner boundary

Structuring element rolls along outer boundary

Opening

Closing

Noisy input

Morphological operations
Basic types of structuring elements

Boundary extraction

Hole filling

Connected components

Given point in hole

Hole filled

All holes filled

Given point in hole
Geodesic dilation

Less than or equal to mask

Geodesic erosion

Greater than or equal to mask

Morphological reconstruction by dilation

Opening by reconstruction

Long, vertical strokes
Structuring element: 5x1 pixels

Hole filling

Input image with hole filled

Hole filling

Input image with holes filled
**Border clearing**

- **Marker image**
- **Input image with no objects touching the border**

**Image segmentation**

- **Input**
- **Edges**
- **Segmentation**

**Detection of isolated points**

- **Input**
- **Segmentation**

**Line detection**

- **Input**
- **Double lines**

**Line detection, specific directions**

- **Spatial filters**
  - **Horizontal**
  - **+45°**
  - **Vertical**
  - **-45°**
  - **Negative values**
  - **Threshold**
Edges

Step
Ramp
Roof edge

Ramp edge

Two points
First derivative
Second derivative
One point

Gradient operators

Gradients

Magnitude of gradient vector
Magnitude of vertical gradient
**Gradients**

Smooth image prior to computing gradients. Results in more selective edges.

**Edge detection**

Threshold magnitude of gradient vector

**Thresholding**

Single threshold

Dual threshold

**Noise**

Input

Intensity ramp

Product of input and intensity ramp
Basic global thresholding

Optimum global thresholding

Variable thresholding

Variable thresholding
Variable thresholding

Input  Global thresholding  Local thresholding using moving averages