Today's learning goals

- Define the regular operations on languages
- Prove closure properties of the class of regular languages
- Trace nondeterministic finite automata to determine whether a string is accepted
- Distinguish between an NFA and a DFA
- Explain why nondeterminism can help
- Design NFA to recognize a given language
True/False: each DFA recognizes a unique language.
I.e. if two DFA are different (different number of states or different initial state, or different transition function, etc.) then they recognize different languages.

A. True
B. False
C. I don't know.

Let $M_1, M_2$ diff't
WTS $L(M_1) \neq L(M_2)$

Counterex: $M_1, M_2$ but
$L(M_1) = L(M_2)$

$\Sigma = \{0, 1, 2\}$

\[\begin{array}{c|ccc}
 & 0 & 1 & 2 \\
\hline
0 & 0 & 1 & 2 \\
1 & 0 & 1 & 2 \\
2 & 0 & 1 & 2 \\
\end{array}\]
Building DFA

Typical questions
e.g. HW2 Q1c, Q2

Define a DFA which recognizes the given language \( L \).

or

Prove that the (given) language \( L \) is regular.
Building DFA

Example

Define a DFA which recognizes

\{ w | w \text{ has at least 2 } a's \}
Building DFA

Example

Define a DFA which recognizes

\{ w | w \text{ has at most 2 } a's \}
Building DFA

Remember

States are our only (computer) memory.

Design and pick states with specific roles / tasks in mind.

"Have not seen any of desired pattern yet"

"Trap state"
The regular operations

For $A$, $B$ languages over same alphabet, define:

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

$$A \circ B = \{xy | x \in A \text{ and } y \in B\}$$

$$A^* = \{x_1 x_2 \ldots x_k | k \geq 0 \text{ and each } x_i \in A\}$$

These are operations on sets!
Closure of ... under ...

- \( \mathbb{Z} \) under addition.
- Set of even ints under multiplication.
- \{0\}* under concatenation.

Which of these is true?

A. The set of odd integers is closed under addition.
B. The set of positive integers is closed under subtraction.
C. The set of rational numbers is closed under multiplication.
D. The set of real numbers is closed under division.
E. I don't know.
Complementation

Claim: If $A$ is a regular language over $\{0,1\}^*$, then so is $A'$

Proof: Let $A$ be regular.
So there's some DFA, $M$, s.t. $L(M) = A$. WTS $\overline{A}$ is regular.
Want to build (new) DFA $M'$ where $L(M') = \overline{A}$. 
\overline{A} = \{ x \mid x \notin A \}.
Claim: If $A$ is a regular language over $\{0,1\}^*$, then so is $\overline{A}$ aka "the class of regular languages is closed under complementation"

Proof: Let $A$ be a regular language. Then there is a DFA $M=(Q,\Sigma,\delta,q_0,F)$ such that $L(M) = A$. We want to build a DFA whose language is $\overline{A}$. Consider

$$M' = (Q,\Sigma,\delta,q_0,F)$$

Claim of Correctness $L(M') = \overline{A}$

Proof of claim…
Why closure proofs?

• Stretch the power of the model
• General technique of proving a new language is regular
• Puzzle!
Theorem: The class of regular languages is closed under the union operation.

Proof:

What are we proving here?

A. For any set A, if A is regular then so is A U A.
B. For any sets A and B, if A U B is regular, then so is A.
C. For two DFAs M1 and M2, M1 U M2 is regular.
D. None of the above.
E. I don't know.
Theorem: The class of regular languages over fixed alphabet $\Sigma$ is closed under the union operation.

Proof: Let $A_1, A_2$ be any two regular languages over $\Sigma$. WTS that $A_1 \cup A_2$ is regular.

Goal: build a machine that recognizes $A_1 \cup A_2$. 
For next time

Homework 2 due next week

- DFA design
- Closure proofs