Turing machines

- Unlimited input
- Unlimited (read/write) memory
- Unlimited time
Formal definition of TM

A Turing machine is a 7-tuple \((Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})\) where \(Q, \Sigma, \Gamma\) are all finite sets and

1. \(Q\) is the set of states
2. \(\Sigma\) is the input alphabet (not containing blank symbol)
3. \(\Gamma\) is the tape alphabet (including blank symbol as well as all symbols in \(\Sigma\))
4. \(\delta : Q \times \Gamma \to Q \times \Gamma \times \{\text{L}, \text{R}\}\) is the transition function
5. \(q_0 \in Q\) is the start state
6. \(q_{\text{accept}} \in Q\) is the accept state
7. \(q_{\text{reject}} \in Q\) is the reject state

\(q_{\text{reject}} \neq q_{\text{accept}}\)
Language of a TM

- $L(M) = \{w \mid M \text{ accepts } w\}$
- $M$ may reject or loop on strings not in $L(M)$
- A language $X$ is **recognizable** if $X=L(M)$ for some TM $M$
- A TM $M$ is a decider if $M(w)$ halts on every input $w$
- A language $X$ is **decidable** is $X=L(M)$ for some decider $M$
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Which of the following is true?

A) If $X$ is decidable, then $X$ is recognizable
B) If $X$ is recognizable, then $X$ is recognizable
C) If $X$ is decidable, then $X$ is decidable
D) I don’t know
TM models

- TM with doubly infinite tape
- TM with 2-dimensional tape
- 2-Tape TM
- K-Tape TM
- Non-deterministic TM
- Theorem: All above models are equivalent
Equivalence between models

- Consider two models, e.g., TM and 2TM
- What does it mean for TM and 2TM to be equivalent?
  - Any TM $M$ can be transformed into a 2TM $M'$ such that $L(M) = L(M')$
  - Any 2TM $M'$ can be transformed into a TM $M$ such that $L(M) = L(M')$
- Strengthen: $M$ terminates iff $M'$ terminates
Church-Turing Thesis

- Theorem: TM, 2TM, k-TM, NTM, etc. are all equivalent
- Theorem: TM, λ-calculus, java, etc. are all equivalent
- Church-Turing thesis: any “reasonable” model of computation is equivalent to the TM
Decidable vs Recognizable

• If A is decidable then A is recognizable
• If A is decidable then \(A\) is recognizable
  - Equivalently, we may say that A is co-recognizable
• Summary: If A is decidable, then A is both recognizable and co-recognizable
• Question: If A is both recognizable and co-recognizable, can be conclude that A is decidable?
Decidable vs Recognizable

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• If A is decidable then \( A \) is recognizable.
  - Equivalently, we may say that \( A \) is co-recognizable.

• Summary: If A is decidable, then A is both recognizable and co-recognizable.

• Question: If A is both recognizable and co-recognizable, can we conclude that A is decidable?

A) Yes  
B) No  
C) It depends on A  
D) I don’t know