Today’s Topics: Equivalence and Validity

1. Proving equivalence of two propositions using truth tables
2. Proving validity of an argument form using truth tables
   - Converse, iff, contrapositive

1. Proving equivalence of two propositions using truth tables

Which pair of propositions are equivalent to each other?

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>¬q</th>
<th>p → q</th>
<th>¬p ∨ q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
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<td>T</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
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</tr>
</tbody>
</table>

A. p, ¬p
B. p → q, ¬p
C. q, ¬q ∨ ¬p
D. p, ¬q
E. Other/none/ more than one
Truth table for \((p \rightarrow q) \land \neg p\)

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>(p \rightarrow q)</th>
<th>(\neg p)</th>
<th>((p \rightarrow q) \land \neg p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
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</table>

\((p \rightarrow q) \land \neg p\) is equivalent to:
A. \(q\)
B. \(p \rightarrow q\)
C. \(\neg p \lor q\)
D. \(\neg p\)
E. Other/none/more than one

Hardware for \((p \rightarrow q) \land \neg p\)

Hardware for \(\neg p\)

Back to the Algebra analogy:
we do similar simplification in jr. high math:
\(a + b - 2a + 3b + a - 8b/2 + a = a\)

Hardware design efficiency

- Improved performance of CPUs depends in part on squeezing logic onto as tiny amount of silicon as possible, and using as little electricity/heat as possible.
- Minimizing logic gates helps both.

How can we prove that two propositions are not equivalent?

A. Make a truth table that has columns for both, and verify that not all the entries in the two columns are \(T\)
B. Make a truth table that has columns for both, and verify that at least one of the entries in the two columns is \(F\)
C. Make a truth table that has columns for both, and make sure that the two columns are not identical to each other.
D. Make a truth table that has columns for both, and make sure that in the row where the input variables are all \(T\), the propositions are both \(F\)
2. Proving validity of an argument form using truth tables
   Also: converse, iff, and contrapositive

Be the fact-checker!

“If this shape is a square, then this shape has four equal-length sides connected at right angles. Therefore, if this shape has four equal-length sides connected at right angles, then this shape is a square.”

Whether or not you agree with the statement, is the argument form valid?

Steps to determining if it is valid:

1. Translate to logic form: \((p \rightarrow q) \Rightarrow (q \rightarrow p)\)
2. Change the “therefore” into “implies”
   - \((p \rightarrow q) \rightarrow (q \rightarrow p)\)
3. Make a truth table for that proposition (preferably step-by-step), and see if it is a tautology

Converse error

Here is another example with the same form:

- If this shape is a square, then this shape is a rectangle. Therefore, if this shape is a rectangle, then this shape is a square.

No!

- \(p \rightarrow q\) and \(q \rightarrow p\) are the converse of each other.
- It is not safe to assume that if \(p \rightarrow q\) is true, then \(q \rightarrow p\) is also true!
- The converse could be true though… as in the equal sides/square example. If both \(p \rightarrow q\) and \(q \rightarrow p\) are true, then we say \(p \leftrightarrow q\) (“p iff q”).
Converse Error

- Here is another argument a similar form. Remember this?
- “If something is made out of wood, then it floats.
- Therefore, if she floats, then she is made out of wood
- [and therefore a witch!]”

Argument validity

What about this?

- If this shape is a square, then this shape is a rectangle. Therefore, if this shape is not a rectangle, then this shape is not a square.

1. Translate to logic, then replace $\Rightarrow$ with $\rightarrow$, giving: $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$
2. Make a truth table:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>$\neg q$</th>
<th>$\neg p$</th>
<th>$p \rightarrow q$</th>
<th>$\neg q \rightarrow \neg p$</th>
<th>$(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$</th>
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Argument form is?

A. Valid
B. Not valid

Why?