1) Use K-maps to simplify the following functions.

   a. \( f(a,b,c) = \prod M(3,4,5) + \sum d (2,7) \)

      i. List all prime implicates,
      ii. List essential prime implicates
      iii. Derive all possible minimal POS (product of sum expressions)

   b. if \( f_1(a,b,c) = \sum m (0,1,5,6) \) and \( f_2(a,b,c) = \sum m (0,1,4) \), draw the k-maps for functions \( f_1, f_2, \) and \( f_1 \& f_2 \) and then derive the minimal SOP representation of function \( f_1 \& f_2 \) (\( f_1 \) and \( f_2 \)).

2) Given \( f(a, b, c, d) = \sum m(0, 3, 4, 5, 10, 14) + \sum d(1, 7) \):

   a. Derive a minimal expression for \( f \)
   b. implement the function using a minimal network of 2:1 multiplexers and minimum number of inverters. Do not use any other logic gates.

3) A museum has three rooms, each with a motion sensor (\( m_0, m_1 \) and \( m_2 \)) that outputs 1 when motion is detected. At night, the only person in the museum is one security guard who walks from room to room. Create a circuit that sounds an alarm (by setting an output A to 1) if motion is ever detected in more than one room at a time (ie in two or three rooms), meaning there must be one or more intruders in the museum

   a. Fill out the truth table
   b. Minimize the expression (use K-maps)
   c. Draw circuit for minimum implementation obtained in part b.

4) Use decoders specified below and a minimum number of other gates to output a 1 for any prime number less than 16.

   a. use minimum number of 3:8 decoders
   b. use minimum number of 2:4 decoders