RULES:

1. Don’t start the exam until the instructor says to.

2. This is a closed-book, closed-notes, no-calculator exam. Don’t refer to any materials other than the exam itself.

3. Write your name, and the login name you are using for the course, on each page of the exam when you get to it. Think carefully about each question. Before you turn in your exam, make sure you have all the pages. The last page is blank and can be used as scratch paper.

4. Do not look at anyone else’s exam. Do not talk to anyone but an exam proctor during the exam. If you’re wearing a billed cap, please turn it around or take it off. And turn off phones and all other devices.

5. If you have a question, raise your hand and an exam proctor will come to you.

6. You have until the end of the class period to finish the exam. When you are done, give your exam to a proctor. The proctor will check your picture ID and sign the ID check below.

7. Your exam grade will be emailed to your account within 3 working days. Exams will be handed back in a session to be announced.

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PROCTOR ID CHECK:__________________
grader:______
1. [21 pts.] (1 pt each correct; -1 pt each incorrect; 0 pt if left blank) True or False:

   a. _____ A treap is a balanced binary tree.
   b. _____ A Huffman code tree is a binary trie.
   c. _____ In a Huffman code tree, every internal node has exactly two children.
   d. _____ In a binary tree with N>0 nodes, there are exactly N-1 non-null child pointers.
   e. _____ An undirected graph is connected if for every pair of distinct vertices V1, V2, there is a path from V1 to V2.
   f. _____ Considered as a binary tree, a heap always satisfies the AVL balance property.
   g. _____ The average number of bits per symbol in a Huffman code for an information source is never less than the entropy of the source.
   h. _____ A precondition of Dijkstra’s algorithm is that the input graph not have a cycle.
   i. _____ Every subtree of a binary trie is itself a binary trie.
   j. _____ Every subtree of a heap is itself a heap.
   k. _____ Data items in a priority queue must be comparable to each other.
   l. _____ The minimum spanning tree problem for undirected graphs is NP complete.
   m. _____ If an undirected graph is not connected, then that graph does not have a spanning tree.
   n. _____ If an undirected graph has a cycle, then that graph does not have a spanning tree.
   o. _____ Any AVL single rotation in a heap preserves the heap ordering property.
   p. _____ C++ allows primitive type (such as int) function arguments to be passed by reference.
   q. _____ A precondition of Kruskal’s algorithm is that the input graph not have negative-weight edges.
   r. _____ C++ allows pointers to primitive type (such as int) variables.
   s. _____ In C++, the function call operator can be overloaded.
   t. _____ In C++, the end() member function of a std::vector object returns an iterator pointing at the last element in the vector.
   u. _____ In C++, “*” is a pointer dereference operator.
2. [15 pts] (3 pts each) Multiple choice. Write the capital letter of the best answer in the blank provided. Assume that the length of a path in an unweighted graph is the number of edges on the path.

a. _____ The maximum possible length of a shortest path in an unweighted graph with 7 vertices is
   A. 0      B. 1      C. 6      D. 7      E. 49

b. _____ The minimum number of edges in a spanning tree for a connected undirected graph with 10 vertices is
   A. 0      B. 1      C. 9      D. 10     E. 45

c. _____ The number of bytes in a Huffman code for a message consisting of 8 distinct symbols of equal probability is
   A. 1      B. 2      C. 3      D. 8      E. 24

d. _____ The maximum possible Huffman code length (in bits) for a symbol drawn from an alphabet of 3 symbols is
   A. 2      B. 3      C. 4      D. 5      E. undetermined

e. _____ Suppose in an input file X appears with probability 0.4, Y appears with probability 0.2, and Z appears with probability 0.4. Which of the following cannot be a Huffman code for the sequence XYYZXY:
   A. 100001100  B. 1011101011  C. 00010110001  D. 0101000010  E. 0101011010
3. [12 pts] Consider the following adjacency matrix for an unweighted directed graph G (blank entries mean no edge):

```
  1  2  3  4  5  6
1  |   |   | 1 |   |   |
2  |   |   |   |   |   |   |
3  |   |   |   |   |   |   |
4  | 1 |   | 1 | 1 |   |   |
5  |   | 1 |   |   |   |   |
6  |   |   |   |   |   | 1 |
```

[6 pts] A breadth-first search starting from vertex 6 will visit vertices in G in some sequence. In fact, several distinct sequences are possible in this case. Show four of them:

___, ____, ____, ____, ____, ____
___, ____, ____, ____, ____, ____
___, ____, ____, ____, ____, ____
___, ____, ____, ____, ____, ____

4. [8 pts] (In this problem keys are characters, with alphabetic ordering; priorities are integers, with larger integers representing higher priority.)

The keys A, B, C, D, E, F, G were inserted, in that order, into an initially empty randomized search tree. The integer priorities generated by the implementation happened to produce a treap which has the smallest average successful-find time cost for any treap containing those keys (assuming all keys are equally likely to be searched for).

a. Write distinct integer priorities associated with each of the 7 keys that could have produced that result:

(A,______), (B,______), (C,______), (D,______),(E,______), (F______), (G______)

b. Draw the treap with those keys and priorities. Clearly label each node with the (key,priority) pair in it:
5. [14 pts. total] Suppose in observing 300 symbols emitted from an information source you count the symbols A,D,E,N,T occurring the number of times shown:

- A: 90
- D: 50
- E: 30
- N: 60
- T: 70

   a. [6 pts] Use Huffman’s algorithm to construct a Huffman code tree for this information source. In constructing this tree, when joining two trees, make the one with the smaller count in its root be the ‘0’ child of its parent; break any ties arbitrarily. In the space above, draw the tree. Label each leaf with the symbol in that leaf node. Label links to children as 1 or 0 as required.

   b. [4 pts] Write down the sequence of symbols coded by this sequence of bits:

      1 1 0 0 1 0 1 0 1 1 0 0 0 0

      ____,____,____,____,____

   c. [4 pts] According to this code, write down the left-to-right sequence of bits that codes this message: TANNED

      (The bits may not completely fill the 2 bytes shown.)

      _______________________________