Photometric Stereo

Computer Vision I
CSE252A
Lecture 7

Announcements
• Read Chapter 2 of Forsyth & Ponce
• Instructor office hours
  • Tuesdays 6:30 PM-7:30 PM
  • Library (for now)
• Homework 1 is due today by 11:59 PM

Shading reveals 3-D surface geometry

Two shape-from-X methods that use shading
• Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.
• Photometric stereo: Single viewpoint, multiple images under different lighting.

Photometric Stereo Rigs:
One viewpoint, changing lighting

An example of photometric stereo
Multi-view stereo vs. Photometric Stereo: Assumptions

- Multi-view Stereo
  - Multiple images
  - Dynamic scene
  - Multiple viewpoints
  - Fixed lighting
- Photometric Stereo
  - Multiple images
  - Static scene
  - Fixed viewpoint
  - Multiple lighting conditions

Photometric stereo

- Single viewpoint, multiple images under different lighting.
  1. Arbitrary known BRDF, known lighting
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting.

I. Photometric Stereo: General BRDF and Reflectance Map

BRDF

- Bi-directional Reflectance Distribution Function
  \[ p(\theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) \]
- Function of
  - Incoming light direction: \( \theta_{\text{in}}, \phi_{\text{in}} \)
  - Outgoing light direction: \( \theta_{\text{out}}, \phi_{\text{out}} \)
- Ratio of incident irradiance to emitted radiance

Coordinate system

Surface: \( s(x, y) = (x, y, f(x, y)) \)
Tangent vectors:
\[
\begin{align*}
\frac{\partial s(x, y)}{\partial x} &= (1, 0, f_x) \\
\frac{\partial s(x, y)}{\partial y} &= (0, 1, f_y)
\end{align*}
\]
Normal vector \( n = \frac{\partial f}{\partial x} \times \frac{\partial f}{\partial y} = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1 \right) \)

Gradient Space (p,q)

Gradient Space: \( (p,q) \)
\[
p = \frac{\partial f}{\partial x} \quad q = \frac{\partial f}{\partial y}
\]
Normal vector \( n = \frac{1}{\sqrt{p^2 + q^2 + 1}} (p, q, -1) \)
For a given point A on the surface, the image irradiance $E(x,y)$ is a function of:
1. The BRDF at A
2. The surface normal at A
3. The direction of the light source

Let the BRDF be the same at all points on the surface, and let the light direction $s$ be a constant.
1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have $E(p,q)$.

Example Reflectance Map: Lambertian surface

What does the intensity (irradiance) of one pixel in one image tell us?
It constrains the surface normal projecting to that point to a curve

A third image would disambiguate match

Two Light Sources
Two reflectance maps

LAMBERTIAN REFLECTANCE MAP
$E = L \rho \frac{1 - \rho \cdot \rho}{\sqrt{1 + \rho^2 - \rho^2 \cdot \rho^2}}$
Three Source Photometric stereo:

Step 1

Offline:
Using source directions & BRDF, construct reflectance map for each light source direction. \( R_1(p,q), R_2(p,q), R_3(p,q) \)

Online:
1. Acquire three images with known light source directions. \( E_1(x,y), E_2(x,y), E_3(x,y) \)
2. For each pixel location \((x,y)\), find \((p,q)\) as the intersection of the three curves
   \[ R_1(p,q)=E_1(x,y) \]
   \[ R_2(p,q)=E_2(x,y) \]
   \[ R_3(p,q)=E_3(x,y) \]
3. This is the surface normal at pixel \((x,y)\). Over image, the normal field is estimated

Plastic Baby Doll: Normal Field

Next step:
Go from normal field to surface

Recovering the surface \( f(x,y) \)

Many methods: Simplest approach
1. From estimate \( n = (n_x, n_y, n_z) \), \( p = n_x/n_z \), \( q = n_y/n_z \)
2. Integrate \( p = df/dx \) along a row \((x,0)\) to get \( f(x,0) \)
3. Then integrate \( q = df/dy \) along each column starting with value of the first row

What might go wrong?

• Height \( z(x,y) \) is obtained by integration along a curve from \((x_0, y_0)\).
  \[ z(x,y) = z(x_0, y_0) + \int_{(x_0, y_0)}^{(x,y)} (pdx + qdy) \]
• If one integrates the derivative field along any closed curve, one expects to get back to the starting value.
• Might not happen because of noisy estimates of \((p,q)\)
What might go wrong?

Integrability. If $f(x,y)$ is the height function, we expect that

$$\frac{\partial f}{\partial y} \frac{\partial}{\partial y} = \frac{\partial f}{\partial x} \frac{\partial}{\partial x}$$

In terms of estimated gradient space $(p,q)$, this means:

$$\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}$$

But since $p$ and $q$ were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold.

Horn's Method

[Horn’s Method]Robot Vision, B.K.P. Horn, 1986]

- Formulate estimation of surface height $z(x,y)$ from gradient field by minimizing cost functional:

$$\int \left( z_x - p \right)^2 + \left( z_y - q \right)^2 \, dx \, dy$$

where $(p,q)$ are estimated components of the gradient while $z_x$ and $z_y$ are partial derivatives of best fit surface.

- Solved using calculus of variations – iterative updating.
- $z(x,y)$ can be discrete or represented in terms of basis functions.
- Integrability is naturally satisfied.

II. Photometric Stereo:
Lambertian Surface, Known Lighting

Lambertian Photometric stereo

- If the light sources $s_1$, $s_2$, and $s_3$ are known, then we can recover $b$ from as few as three images. (Photometric Stereo: Silver 80, Woodham81).

$$\begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix} = b^T \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}$$

- i.e., we measure $e_1$, $e_2$, and $e_3$ and we know $s_1$, $s_2$, and $s_3$. We can then solve for $b$ by solving a linear system.

$$b^T = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix} \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^{-1}$$

- Normal is: $n = b/|b|$, albedo is: $|b|$.

What if we have more than 3 Images?
Linear Least Squares

Let the residual be $r = e - Sb$

Squaring this:

$$r^T = r^T (e - Sb)^T (e - Sb)$$

$$= e^T e - 2b^T S^T e + b^T S^T S b$$

- Zero derivative is a necessary condition for a minimum, or

$$-2S^T e + 2S^T S b = 0$$

Solving for $b$ gives

$$b = (S^T S)^{-1} S^T e$$
An example of photometric stereo

III. Photometric Stereo with unknown lighting and Lambertian surfaces

Covered in Illumination cone slides