Image Formation and Cameras

Computer Vision I
CSE 252A
Lecture 3

Announcements

• http://cseweb.ucsd.edu/classes/fa14/cse252A-b/
• Piazza
• Course reserves available
• Instructor office hours TBD
• Homework 0 is due today by 11:59 PM
• Wait list
• Read:
  – Chapters 1 & 2 of Forsyth & Ponce
  – Chapter 2 of Szeliski (Optional)

Image Formation: Outline

• Factors in producing images
• Projection
• Perspective/Orthographic Projection
• Vanishing points
• Projective Geometry
• Rigid Transformation and SO(3)
• Lenses
• Sensors
• Quantization/Resolution
• Illumination
• Reflectance and Radiometry

Earliest Surviving Photograph

• First photograph on record, “la table service” by Nicephore Niepce in 1822.
• Note: First photograph by Niepce was in 1816.

Compare to Paintings

Willem Kalf, Mid 1600’s
Pedro Campos,

How Cameras Produce Images

• Basic process:
  – photons hit a detector
  – the detector becomes charged
  – the charge is read out as brightness

• Sensor types:
  – CCD (charge-coupled device)
    • high sensitivity
    • high power
    • cannot be individually addressed
    • blooming
  – CMOS
    • simple to fabricate (cheap)
    • lower sensitivity, lower power
    • can be individually addressed
Images are two-dimensional patterns of brightness values. They are formed by the projection of 3D objects.

Effect of Lighting: Monet

Change of Viewpoint: Monet

Haystack at Chailly at sunrise (1865)

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

Camera Obscura

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays". — Leonardo Da Vinci

Camera Obscura

- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

http://www.acmi.net.au/AIC/CAMERA_OBSCUARA.html (Russell Naughton)
Distant objects are smaller

Purely Geometric View of Perspective

Geometric properties of projection

Equation of Perspective Projection

A Digression

Projective Geometry
and
Homogenous Coordinates
What is the intersection of two lines in a plane?

A Point

Do two lines in the plane always intersect at a point?

No, Parallel lines don’t meet at a point.

Can the perspective image of two parallel lines meet at a point?

YES

Projective geometry provides an elegant means for handling these different situations in a unified way, and homogenous coordinates are a way to represent entities (points & lines) in projective spaces.

Projective Geometry

- Axioms of Projective Plane
  1. Every two distinct points define a line
  2. Every two distinct lines define a point (intersect at a point)
  3. There exists three points, A,B,C such that C does not lie on the line defined by A and B.
- Different than Euclidean (affine) geometry
- Projective plane is “bigger” than affine plane – includes “line at infinity”

Homogenous coordinates

A way to represent points in a projective space

- Use three numbers to represent a point on a projective plane
  Why? The projective plane has to be bigger than the Cartesian plane.

  How: Add an extra coordinate
  e.g., \((x,y) \rightarrow (x,y,1)\)

  Impose equivalence relation
  \((x,y,1) = \lambda (x,y,2)\)

  i.e., \((x,y,1) = (\lambda x, \lambda y, \lambda)\)

- Possible to represent points “at infinity”
  - Where parallel lines intersect
  - Where parallel planes intersect
  - Possible to write the action of a perspective camera as a matrix

  • Point at infinity – zero for last coordinate
  e.g., \((x,y,0)\)
Homogenous coordinates
A way to represent points in a projective space
Use three numbers to represent a point on a projective plane
Add an extra coordinate
e.g., (x,y) -> (x,y,1)
Impose equivalence relation
\((x,y,z) \sim \lambda \cdot (x,y,z)\)
such that \((\lambda \neq 0)\)
i.e., \((x,y,1) \sim (\lambda x, \lambda y, \lambda)\)

Conversion
Euclidean -> Homogenous -> Euclidean
In 2-D
- Euclidean -> Homogenous: \((x, y) \rightarrow k (x,y,1)\)
- Homogenous -> Euclidean: \((x, y, z) \rightarrow (x/z, y/z)\)
In 3-D
- Euclidean -> Homogenous: \((x, y, z) \rightarrow k (x,y,z,1)\)
- Homogenous -> Euclidean: \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)

Points at infinity
Point at infinity – zero for last coordinate \((x,y,0)\)
and equivalence relation
\((x,y,0) \sim \lambda \cdot (x,y,0)\)
No corresponding Euclidean point

Lines in Projective space
1. Line in Euclidean plane
2. Plane through origin in homogenous coordinates
3. Plane is represented by its normal \(N\)
4. Equation for plane is \(N \cdot (x,y,z) = 0\)
or \(M \cdot (x,y,z) = 0\)
where \(M = \lambda N\)

Projective transformation
- 3 x 3 linear transformation of homogenous coordinates
- Points map to points,
  lines map to lines
\[
\begin{pmatrix}
  u_1 \\
u_2 \\
u_3
\end{pmatrix} = \begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
\lambda & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{pmatrix}
\begin{pmatrix}
  x_1 \\
x_2 \\
x_3
\end{pmatrix}
\]

The equation of projection
Cartesian coordinates:
\((x,y,z) \rightarrow (x^2, y^2, z^2)\)
Homogenous Coordinates
and Camera matrix
\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix}
d & 0 & 0 \\
0 & g & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix}
\]
\[
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix} = \begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\begin{pmatrix}
r & 0 & 0 \\
0 & r & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x'' \\
y'' \\
z''
\end{pmatrix}
\]
\[
\begin{pmatrix}
x'' \\
y'' \\
z''
\end{pmatrix} = \begin{pmatrix}
x'' \\
y'' \\
z''
\end{pmatrix}
\begin{pmatrix}
r & 0 & 0 \\
0 & r & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x''' \\
y''' \\
z'''
\end{pmatrix}
\]
End of the Digression

Parallel lines meet in the image

Vanishing point

Image plane

- Formed by line through $O$
- Parallel to the given line(s)
- A single line can have a vanishing point

Vanishing points

Different directions correspond to different vanishing points

Vanishing Point

- In the **projective plane**, parallel lines meet at a point at infinity.
- The vanishing point is the perspective projection of that point at infinity, resulting from multiplication by the camera matrix.

Projective transformation

- $3 \times 3$ linear transformation of homogenous coordinates
- Points map to points,
- lines map to lines

$$
\begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
$$
Mapping from a Plane to a Plane under Perspective is given by a projective transform $H$

$$X' = Hx$$

Planar Homography

$$x' = H_1X = H_2(H_1^{-1}x) = (H_2H_1^{-1})x$$

Planar Homography: Pure Rotation

Application: Panoramas