Next

• More on recursion

• Higher-order functions
  - taking and returning functions

• Along the way, will see map and fold
max function

let max x y = if x < y then y else x;;

(* return max element of list l *)
let list_max l =
let max x y = if x < y then y else x;;

(* return max element of list l *)
let list_max l =
    let rec helper curr l =
        match l with
        | [] -> curr
        | h::t -> helper (max h curr) t
    in
    helper 0 l;;
concat function

(* concatenate all strings in a list *)
let concat l =
concat function

(* concatenate all strings in a list *)

let concat l =
    let rec helper curr l =
        match l with
        [] -> curr
        | h::t -> helper (curr ^ h) t
    in
    helper "" l;;
What’s the pattern?

```ocaml
let list_max l =  
  let rec helper curr l =  
    match l with  
      [] -> curr  
    | h::t -> helper (max h curr) t  
  in helper 0 l;;
```

```ocaml
let concat l =  
  let rec helper curr l =  
    match l with  
      [] -> curr  
    | h::t -> helper (curr ^ h) t  
  in helper "" l;;
```
fold, the general helper func!

(* to help us see the pattern: *)
let list_max l =
  let rec helper curr l =
    match l with
    | [] -> curr
    | h::t -> helper (max h curr) t
  in helper 0 l;;

(* fold, the coolest function there is! *)
let rec fold f curr l =
(* fold, the coolest function there is! *)

let rec fold f curr l =
    match l with
    [] -> curr
    | h::t -> fold f (f h curr) t;;
let rec fold f curr l =
    match l with
    | []  -> curr
    | h::t -> fold f (f h curr) t;;
Examples of fold

```haskell
let list_max =

let concat =

let multiplier =
```
Examples of fold

``` OCaml 
let list_max = fold max 0;;

let concat = fold (^) "";;

let multiplier = fold (*) 1;;
```
Examples of fold

```
let fact n =
    multiplier (interval 1 n);
```

Notice how all the recursion is buried inside two functions: interval and fold!
Examples of fold

let cons x y = x::y;;
let f = fold cons [];;
(* same as:
   let f l = fold cons [] l *)

[[]] 🔄 [v₁] 🔄 f [v₂;v₃] 🔄 [v₄]
Examples of fold

```ocaml
define cons x y = x::y;
define f = fold cons [];;
(* same as: *
define f l = fold cons [] l *)
```
More recursion: interval

(* return a list that contains the integers i through j inclusive *)
let rec interval i j =
(* return a list that contains the integers i through j inclusive *)

let rec interval i j =
  if i > j
  then []
  else i::(interval (i+1) j);;
interval function with init fn

(*) return a list that contains the elements f(i), f(i+1), ..., f(j) *)
let rec interval_init i j f =
interval function with init fn

(* return a list that contains the elements f(i), f(i+1), ... f(j) *)

let rec interval_init i j f =
    if  i > j
    then []
    else (f i)::(interval_init (i+1) j f);;
interval function again

(* our regular interval function in terms of the one with the init function *)
let rec interval i j =
interval function again

(* our regular interval function in terms of the one with the init function *)
let rec interval i j =
interval_init i j (fun x -> x);;
Interval function yet again!

(* let's change the order of parameters... *)
let rec interval_init f i j =
  if  i > j
  then []
  else (f i)::(interval_init f (i+1) j);;

(* now can use currying to get interval function! *)
let interval = interval_init (fun x -> x);;
Function Currying

In general, these two are equivalent:

```ml
let f = fun x1 -> ... -> fun xn -> e
```

```ml
let f x1 ... xn = e
```

Multiple argument functions by returning a function that takes the next argument
• Named after a person (Haskell Curry)
# Function Currying vs tuples

<table>
<thead>
<tr>
<th>Tuple version:</th>
<th>fn definition</th>
<th>fn call</th>
</tr>
</thead>
<tbody>
<tr>
<td>let f (x₁,...,xₙ) = e</td>
<td>f (x₁,...,xₙ)</td>
<td></td>
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</tbody>
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<td></td>
</tr>
</tbody>
</table>
Function Currying vs tuples

Consider the following:

```ml
let lt x y = x < y;
```

Could have done:  

```
let lt (x,y) = x<y;
```

- But then no “testers” possible

In general: Currying allows you to set just the first n params (where n smaller than the total number of params)
map

(* return the list containing f(e) for each element e of l *)

let rec map f l =
(* return the list containing f(e) for each element e of l *)

let rec map f l =
    match l with
    | [] -> []
    | h::t -> (f h)::(map f t);;
let incr x = x+1;;

let map_incr = map incr;;
map_incr (interval (-10) 10);;
composing functions

\[(f \circ g) (x) = f(g(x))\]

(* return a function that given an argument x applies f2 to x and then applies f1 to the result*)

let compose f1 f2 =
composing functions

\[(f \circ g) (x) = f(g(x))\]

(* return a function that given an argument x applies f2 to x and then applies f1 to the result*)

let compose f1 f2 = fun x -> (f1 (f2 x));;

(* another way of writing it *)

let compose f1 f2 x = f1 (f2 x);;
Higher-order functions!

```ml
let map_incr_2 = compose map_incr map_incr;;
map_incr_2 (interval (-10) 10);;

let map_incr_3 = compose map_incr map_incr_2;;
map_incr_3 (interval (-10) 10);;

let map_incr_3_pos = compose pos_filer map_incr_3;;
map_incr_3_pos (interval (-10) 10);;
(compose map_incr_3_pos pos_filer) (interval (-10) 10);;
```
Higher-order functions!

Instead of manipulating lists, we are manipulating the list manipulators!
Exercise 1

```
let rec filter f l =  
  match l with  
  | [] -> []  
  | h::t -> let t' = filter f t in  
    if f h then h::t' else t'

let neg f x = not (f x)

let partition f l = (filter f l, filter (neg f) l)
```

This implementation is not ideal, since it unnecessarily processes the list twice. Rewrite partition so that it is a single call to fold_left, so the input list is processed only once. Recall:

```
val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a
```
Exercise 1

val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a

let partition f l =
Exercise 1 Solution

```ocaml
val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a

let partition f l =
  let fold_fn (pass,passnot) x =
    if f x then (pass@[x], passnot)
    else (pass, passnot@[x])
  in
  List.fold_left fold_fn ([],[]) l;;
```
Exercise 2

val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a
val map : ('a -> 'b) -> 'a list -> 'b list

Implement map using fold:

let map f l =
Exercise 2 Solution

val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a
val map : ('a -> 'b) -> 'a list -> 'b list

Implement map using fold:

let map f l =
  List.fold_left (fun acc x -> acc@[f x]) [] l
Benefits of higher-order functions

Identify common computation “patterns”

- **Iterate** a function over a set, list, tree ...
- **Accumulate** some value over a collection

Pull out (factor) “common” code:
- **Computation Patterns**
- **Re-use** in many different situations
Funcs taking/returning funcs

Higher-order funcs enable **modular code**

• Each part only needs **local** information

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**Data Structure**

**Client**

Uses **list**

Uses meta-functions:

map, fold, filter

With locally-dependent funs:

 lt h, square etc.

Without requiring Implement.

details of data structure

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**Data Structure**

**Library**

**list**

Provides meta-functions:

map, fold, filter

to traverse, accumulate over lists, trees etc.

Meta-functions don’t need client info
Different way of thinking

“Free your mind”
-Morpheus

- Different way of thinking about computation
- Manipulate the manipulators