Problem 1  Show that the class of decidable (i.e., recursive) languages is closed under Kleene star.

   Hint: There’s an elegant algorithm, using dynamic programming, that makes $O(n^2)$ oracle calls when checking a string of length $n$; you might wish to describe this algorithm instead of the naïve exponential one.

Problem 2  Show that the class of R.E. (i.e., recognizable) languages is closed under intersection.

Problem 3  Consider a variant definition of Turing machines, called always-right Turing machines. In such a machine, the transition function $\delta: Q \times \Gamma \to Q \times \Gamma \times \{R\}$ specifies that the machine always moves its head right (“R”). There is no way for a machine in the always-right model to move its head left.

Show that the always-right Turing machine model is not equivalent to the Turing machine model defined in class and in Sipser.

   Hint: Always-right Turing machines are, in fact, equivalent in power to DFAs.

Problem 4  Let $L_4$ be the language

\[ \{ \langle M, w \rangle \mid M \text{ is a Turing machine, } w \text{ is a string, and } M \text{ never moves its head left when run on input } w. \} \]

Show that $L_4$ is decidable (i.e., recursive), by describing a Turing machine that decides $L_4$.

(Note that the Turing machines here are the ordinary Turing machines defined in class and in Sipser, not the always-right machines of problem 3, and their transition function is allowed to specify a move left.)

Problem 5  Let $L_5$ be the language

\[ \{ \langle G, D \rangle \mid G \text{ is a CFG, } D \text{ is a DFA, and } L(D) \subseteq L(G). \} \]

Show that $L_5$ is undecidable.

Is $L_5$ R.E.? Is it co-R.E.?