Problem 1 For a language $L$ over an alphabet $\Sigma$, define the language $\text{THREES}(L)$ as follows:

$$\text{THREES}(L) = \{abc \mid a, b, c \in \Sigma \text{ and } \exists x \in \Sigma^* \text{ s.t. } abcx \in L\}$$

Note that every string in $\text{THREES}(L)$ is exactly three characters long.

a. Prove that whenever $L$ is a regular language so is $\text{THREES}(L)$. (That is, show that the class of regular languages is closed under the operation $\text{THREES}$.)

b. In hw3.hs, give a definition, including a type signature, for a function $\text{threes}$ that takes a DFA for a language $L$ and returns a DFA or an NFA for $\text{THREES}(L)$. Note that you will need to pick what type the resulting machine’s states will be drawn from.

Problem 2 For a language $L$ over an alphabet $\Sigma$, define the language $\text{HasPrefix}(L)$ as follows:

$$\text{HasPrefix}(L) = \{xy \mid x, y \in \Sigma^*, x \in L, xy \in L, \text{ and } |y| \geq 0\}$$

Note the last condition, which requires that $y$ be nonempty.

a. Prove that whenever $L$ is a regular language so is $\text{HasPrefix}(L)$. (That is, show that the class of regular languages is closed under the operation $\text{HasPrefix}$.)

b. In hw3.hs, give a definition, including a type signature, for a function $\text{hasprefix}$ that takes a DFA for a language $L$ and returns a DFA or an NFA for $\text{HasPrefix}(L)$. Note that you will need to pick what type the resulting machine’s states will be drawn from.