Radiometry and Reflectance

Computer Vision I
CSE 252A
Lecture 7a

Announcements
• HW1 assigned.

Solid Angle
• By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point
• The solid angle subtended by a patch area dA is given by
  \[ d\omega = \frac{dA \cos \theta}{r^2} \]

Radiance
• Power is energy per unit time
• Radiance: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle
• Symbol: \( L(x, \theta, \phi) \)
• Units: watts per square meter per steradian: \( \text{w/(m}^2\text{sr}) \)

Irradiance
• How much light is arriving at a surface?
• Units of irradiance: \( \text{Watts/m}^2 \)
• This is a function of incoming angle.
• A surface experiencing radiance \( L(x, \theta, \phi) \) coming in from solid angle \( d\omega \) experiences irradiance:
  \[ E(x) = L(x, \theta, \phi) \cos \theta d\omega \]

Image Irradiance Equation
\[ E = \frac{\pi}{4} \left( \frac{d}{z} \right)^2 \cos^4 \alpha L \]
Camera’s sensor

- Measured pixel intensity is a function of irradiance integrated over
  - Pixel’s area
  - Over a range of wavelengths
  - For some period of time

\[ I = \int_{\lambda}^{\lambda_2} \int_{y}^{y_2} E(x, y, \lambda, t) s(x, y) q(\lambda) dy dx d\lambda dt \]

- Ideally, it’s linear to the radiance, but the camera response \( C(.) \) may not be linear

\[ I = C \left( \int_{\lambda}^{\lambda_2} \int_{y}^{y_2} E(x, y, \lambda, t) s(x, y) q(\lambda) dy dx d\lambda dt \right) \]

Image sensor

Two types:

1. CCD
2. CMOS

CCD

Separate photo sensor at regular positions
No scanning
Charge-coupled devices (CCDs)

- Interline transfer and frame transfer
- Photosensitive
- Storage

- Each photo sensor has its own amplifier
- More noise (reduced by subtracting ‘black’ image)
- Lower sensitivity (lower fill rate)
- Uses standard CMOS technology
- Allows other components to be put on chip
- ‘Smart’ pixels

CMOS

- Recent technology
- Standard IC technology
- Cheap
- Low power
- Less sensitive
- Per pixel amplification
- Random pixel access
- Smart pixels
- On chip integration with other components

CCD vs. CMOS

- Mature technology
- Specific technology
- High production cost
- High power consumption
- Higher fill rate
- Blooming
- Sequential readout

Color Cameras

We consider 3 concepts:

1. Prism (with 3 sensors)
2. Filter mosaic
3. Filter wheel
- … and X3
**Prism color camera**
Separate light in 3 beams using dichroic prism
Requires 3 sensors & precise alignment
Good color separation

**Filter mosaic**
Coat filter directly on sensor
Demosaicing (obtain full colour & full resolution image)

**Filter wheel**
Rotate multiple filters in front of lens
Allows more than 3 colour bands

Only suitable for static scenes

**Prism vs. mosaic vs. wheel**

<table>
<thead>
<tr>
<th>approach</th>
<th>Prism</th>
<th>Mosaic</th>
<th>Wheel</th>
</tr>
</thead>
<tbody>
<tr>
<td># sensors</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Separation</td>
<td>High</td>
<td>Average</td>
<td>Good</td>
</tr>
<tr>
<td>Cost</td>
<td>High</td>
<td>Low</td>
<td>Average</td>
</tr>
<tr>
<td>Framerate</td>
<td>High</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Artefacts</td>
<td>Low</td>
<td>Aliasing</td>
<td>Motion</td>
</tr>
<tr>
<td>Bands</td>
<td>3</td>
<td>3</td>
<td>3 or more</td>
</tr>
</tbody>
</table>

High-end cameras | Low-end cameras | Scientific applications

**“newer” color CMOS sensor**
Foveon’s X3 – Sigma, Fujifilm

<table>
<thead>
<tr>
<th>better image quality</th>
<th>smarter pixels</th>
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Better image quality, smarter pixels

**Light at surfaces**

Many effects when light strikes a surface -- could be:
- transmitted
  - Skin, glass
- reflected
  - mirror
- scattered
  - milk
- travel along the surface and leave at some other point
- absorbed
  - sweaty skin

**Assume that**
- surfaces don’t fluoresce
  - e.g. scorpions, detergents
- surfaces don’t emit light (i.e. are cool)
- all the light leaving a point is due to that arriving at that point
BRDF

With assumptions in previous slide
• Bi-directional Reflectance
  Distribution Function
  \( \rho(\theta_{in}; \varphi_{in}; \theta_{out}; \varphi_{out}) \)
• Ratio of emitted radiance to
  incident irradiance (units: \( \text{sr}^{-1} \))
• Function of
  – Incoming light direction:
    \( \theta_{in}, \varphi_{in} \)
  – Outgoing light direction:
    \( \theta_{out}, \varphi_{out} \)

\[
\rho(\xi; \theta_{in}, \varphi_{in}; \theta_{out}, \varphi_{out}) = \frac{L_{o}(\xi; \theta_{in}, \varphi_{in})}{L_{i}(\xi; \theta_{in}, \varphi_{in}) \cos \theta_{in}} \sin \vartheta_{in} d\vartheta_{in}
\]

Where \( \rho \) is sometimes denoted \( f \).

Properties of BRDFs

1. Non-negative: \( \rho(\theta_{in}, \varphi_{in}; \theta_{out}, \varphi_{out}) \geq 0 \)
2. Helmholtz Reciprocity Principle:
   \( \rho(\theta_{in}, \varphi_{in}; \theta_{out}, \varphi_{out}) = \rho(\theta_{out}, \varphi_{out}; \theta_{in}, \varphi_{in}) \)
3. Total energy leaving a surface must be less than total
   energy arriving at the surface

\[
\int_{N} L_{i}(x, \omega_{i}) \cos \theta_{i} d\omega_{i} = \int_{N} \int \rho(\theta_{i}, \varphi_{i}; \theta_{in}, \varphi_{in}) L_{o}(x, \omega_{o}) \cos \theta_{o} d\omega_{o}
\]

Surface Reflectance Models

Common Models
• Lambertian
• Phong
• Physics-based
  – Diffuse [Hannahan, Kreuger 1993]
  – Generalized Lambertian [Oren, Nayar 1995]
  – Thoroughly Pitted Surfaces [Koenderink et al 1999]
• Phenomenological
  – [Koenderink, Van Doorn 1996]

 Arbitrary Reflectance
• Non-parametric model
• Anisotropic
• Non-uniform over surface
• BRDF Measurement [Dana et al, 1999], [Marschner 2000]

Specialized
• Hair, skin, threads, paper [Jensen et al]

Important class of BRDF’s: Isotropic BRDF

\( f(\theta_{o}, \varphi_{o}, \theta_{i}, \varphi_{i}) = f(\theta_{o}, \theta_{i}; \varphi_{i} - \varphi_{o}) \)

Isotropic BRDF’s are symmetric about the surface normal. If the surface is rotated about the normal for the same incident and emitting directions, the value of the BRDF is the same.
Lambertian (Diffuse) Surface

- BRDF is a constant called the albedo. $p(\Omega_{in}, \theta_i, \phi_i, \theta_o, \phi_o) = K$
- Emitted radiance is NOT a function of outgoing direction – i.e. constant in all directions.
- For lighting coming in single direction $\omega_i$, emitted radiance is proportional to cosine of the angle between normal and light direction

$$L_e = KN \cdot \omega_i$$

Lambertian reflection

Specular Reflection: Smooth Surface

Rough Specular Surface

Phong Model

Non-Lambertian reflectance
General BRDF: e.g. Velvet

[ After Koenderink et al. 1998 ]

Ways to measure BRDFs

Gonioreflectometers

- Three degrees of freedom spread among light source, detector, and/or sample

Ward’s BRDF Measurement Setup

- Collect reflected light with hemispherical (should be ellipsoidal) mirror [SIGGRAPH 92]

Gonioreflectometers

- Can add fourth degree of freedom to measure anisotropic BRDFs
Ward’s BRDF Measurement Setup

- Result: each image captures light at all exitant angles

Marschner’s Image-Based BRDF Measurement

- For uniform BRDF, capture 2-D slice corresponding to variations in normals

BRDF Not Always Appropriate

- How bright (or what colour) are objects?
- One more definition: Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
- Similar to irradiance

Light sources and shading

- How bright (or what colour) are objects?
- One more definition: Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
- Similar to irradiance

Radiosity due to a point source

- Radiosity is
  \[
  b(r) = \pi \rho_s \langle T \rangle \cos \theta \, d\omega
  = \rho_s \langle T \rangle \cos \theta \, d\omega
  = \rho_s \langle T \rangle \pi \sin \theta \, d\phi
  = \rho_s \langle T \rangle \sin \theta \, d\phi
  \]

- small, distant sphere radius \( \varepsilon \) and exitance \( E \), which is far away subtends solid angle of about \( \Theta \)
Standard nearby point source model

\[ \rho_d(x) \left( \frac{N(x) \cdot S(x)}{r(x)} \right) \]

- N is the surface normal
- \( \rho \) is diffuse (Lambertian) albedo
- S is source vector - a vector from x to the source, whose length is the intensity term
- works because a dot-product is basically a cosine

Area sources

- Examples: diffuser boxes, white walls.
- The radiosity at a point due to an area source is obtained by adding up the contribution over the section of view hemisphere subtended by the source
  - change variables and add up over the source
- See Forsyth & Ponce or a graphics text for details.

Standard distant point source model

- Issue: nearby point source gets bigger if one gets closer
  - the sun doesn’t for any reasonable meaning of closer
- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn’t vary much, and the distance doesn’t vary much either, and we can roll the constants together to get:

Lighting at infinity

- Direction is a three vector s, with |s| = 1.
- Described as function on a sphere: radiance as a function of direction \( r(x) \)
- Single point source is a delta function at some direction
- Multiple point sources: sum of delta functions

Diffuse lighting at infinity: Spherical Harmonics

- \( Y_{lm}(\Theta, \Phi) \)
- Green: Positive
- Blue: Negative

\( l=0 \)
\( l=1 \)
\( l=2 \)

\( m = -2 \quad m = -1 \quad m = 0 \quad m = 1 \quad m = 2 \)
From Leonard McMillan’s SIGGRAPH 99 course notes

Conversely, the light emitted at a given point also is a function on a 2-D space (sphere).

Conversely, the set of light rays emitted from all points...

\[ p = P(\theta, \phi, x, y, z, \lambda, t) \]

Light Field/Lumigraph Main Idea

- In free space, the 5-D plenoptic function can be reduced to a 4-D function (radiances) on the space of light rays.
- Camera images measure the radiance over a 2-D set – a 2-D subset of the 4-D light field.
- Rendered images are also a 2-D subset of the 4-D lumigraph.
- Likewise, the space of emitted lighting can be 4-D.
- Special lights like point source, line sources, area sources are special cases.

Shadows cast by a point source

- A point that can’t see the source is in shadow
- For point sources, the geometry is simple

Area Source Shadows

1. Fully illuminated
2. Penumbra
3. Umbra (shadow)

Shading models

Local shading model
- Surface has incident radiance due only to sources visible at each point
- Advantages:
  - often easy to manipulate, expressions easy
  - supports quite simple theories of how shape information can be extracted from shading
- Used in vision & real-time graphics

Global shading model
- Surface radiosity is due to radiance reflected from other surfaces as well as from surfaces
- Advantages:
  - usually very accurate
- Disadvantage:
  - extremely difficult to infer anything from shading values
- Rarely used in vision, often in photorealistic graphics
A view of a black room, under bright light. Below, we see a cross-section of the image intensity corresponding to the line drawn on the image.

What’s going on here?

- local shading model is a poor description of physical processes that give rise to images
  - because surfaces reflect light onto one another
- This is a major nuisance; the distribution of light (in principle) depends on the configuration of every radiator; big distant ones are as important as small nearby ones (solid angle)
- The effects are easy to model
- It appears to be hard to extract information from these models

At the top, geometry of a gutter with triangular cross-section; below, predicted radiosity solutions, scaled to lie on top of each other for different albedos of the geometry. When albedo is close to zero, shading follows a local model; when it is close to one, there are substantial reflexes.

Irradiance observed in an image of this geometry for a real white gutter.