Announcements

- Assignment 0: “*Getting Started with Matlab*” is due Today.
- Read Chapters 1 & 2 of Forsyth & Ponce
- Wait list + Extension
- Office Hours; Tuesday 4:30-5:30 (?)

Image Formation: Outline

- Factors in producing images
- Projection
- Perspective/Orthographic Projection
- Vanishing points
- Projective Geometry
- Rigid Transformation and SO(3)
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance and Radiometry

The course

- Part 1: The physics of imaging
- Part 2: Early vision
- Part 3: Reconstruction
- Part 4: Recognition

Earliest Surviving Photograph

- First photograph on record, “la table service” by Nicephore Niepce in 1822.
- Note: First photograph by Niepce was in 1816.

Compare to Paintings

*Willem Kalf, Mid 1600’s*  
*Pedro Campos,*
How Cameras Produce Images

- Basic process:
  - photons hit a detector
  - the detector becomes charged
  - the charge is read out as brightness

- Sensor types:
  - CCD (charge-coupled device)
    - high sensitivity
    - high power
    - cannot be individually addressed
    - blooming
  - CMOS
    - simple to fabricate (cheap)
    - lower sensitivity, lower power
    - can be individually addressed

Images are two-dimensional patterns of brightness values.

Effect of Lighting: Monet

Change of Viewpoint: Monet

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

Camera Obscura

*"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays"* — Leonardo Da Vinci

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)
Camera Obscura

- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

Jetty at Margate England, 1898.

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

Distant objects are smaller

Purely Geometric View of Perspective

The projection of the point \( P \) on the image plane \( \Pi' \) is given by the point of intersection \( P' \) of the ray defined by \( PO \) with the plane \( \Pi' \)

Equation of Perspective Projection

Cartesian coordinates:
- We have, by similar triangles, that \((x, y, z) \rightarrow (f'x/z, f'y/z, f')\)
- Establishing an image plane coordinate system at \( C' \) aligned with \( i \) and \( j \), we get \((x, y, z) \rightarrow (f'x/z, f'y/z, f')\)

Geometric properties of projection

- 3-D points map to points
- 3-D lines map to lines
- Planes map to whole image or half-plane
- Polygons map to polygons

- Important point to note: Angles & distances not preserved, nor are inequalities of angles & distances.
- Degenerate cases:
  - line through focal point project to point
  - plane through focal point projects to a line
A Digression

Projective Geometry and Homogenous Coordinates

What is the intersection of two lines in a plane?

No, Parallel lines don’t meet at a point.

Can the perspective image of two parallel lines meet at a point?

YES

Projective geometry provides an elegant means for handling these different situations in a unified way, and homogenous coordinates are a way to represent entities (points & lines) in projective spaces.

Projective Geometry

- Axioms of Projective Plane
  1. Every two distinct points define a line
  2. Every two distinct lines define a point (intersect at a point)
  3. There exists three points, A,B,C such that C does not lie on the line defined by A and B.
- Different than Euclidean (affine) geometry
- Projective plane is “bigger” than affine plane – includes “line at infinity”
Homogenous coordinates
A way to represent points in a projective space
- Use three numbers to represent a point on a projective plane
  Why? The projective plane has to be bigger than the Cartesian plane.
How: Add an extra coordinate
e.g., (x,y) -> (x,y,1)
Impose equivalence relation
(x,y,z) ≈ λ*(x,y,z)
such that (λ, not 0)
i.e., (x,y,1) ≈ (λx, λy, λ)
- Point at infinity – zero for last coordinate
e.g., (x,y,0)
Why do this?
- Possible to represent points “at infinity”
  - Where parallel lines intersect
  - Where parallel planes intersect
- Possible to write the action of a perspective camera as a matrix
Conversion
Euclidean -> Homogenous -> Euclidean
In 2-D
- Euclidean -> Homogenous:
  (x, y) -> k (x,y,1)
- Homogenous -> Euclidean:
  (x, y, z) -> (x/z, y/z)
In 3-D
- Euclidean -> Homogenous:
  (x, y, z) -> k (x,y,z,1)
- Homogenous -> Euclidean:
  (x, y, z, w) -> (x/w, y/w, z/w)
Points at infinity
Point at infinity – zero for last coordinate (x,y,0) and equivalence relation
(x,y,0) ≈ λ*(x,y,0)
No corresponding Euclidean point
Lines in Projective space
1. Line in Euclidean plane
2. Plane through origin in homogenous coordinates
3. Plane is represented by its normal N
4. Equation for plane is
   \[ N \cdot (x,y,z) = 0 \]
   or \[ M \cdot (x,y,z) = 0 \]
   where \( M = \lambda N \)
Projective transformation
- 3 x 3 linear transformation of homogenous coordinates
- Points map to points,
  lines map to lines
\[
\begin{bmatrix}
    u_1 \\
    u_2 \\
    u_3
\end{bmatrix} =
\begin{bmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix}
\]
The equation of projection

Cartesian coordinates:

\((x, y, z) \rightarrow (f_x z, f_y z)\)

Homogenous Coordinates and Camera matrix

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & f_x \\
0 & 0 & f_y & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Parallel lines meet in the image

- Formed by line through \(O\)
- Parallel to the given line(s)
- A single line can have a vanishing point

Vanishing Points

- In the projective plane, parallel lines meet at a point at infinity.
- The vanishing point is the perspective projection of that point at infinity, resulting from multiplication by the camera matrix.
Projective transformation

- A 3 x 3 linear transformation of homogeneous coordinates
- Points map to points,
- lines map to lines

\[
\begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 \\
\end{bmatrix} = \begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33} \\
\end{bmatrix} \begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
\end{bmatrix}
\]

Application: OCRs, scan,…

\[x' = Hx\]

Planar Homography

\[x' = H_2X = H_2(H_1^{-1}x) = (H_2H_1^{-1})x\]

Planar Homography: Pure Rotation

Application: Panoramas