MATH CSE20 Homework 7
DUE Monday November 18

Assigned reading: SF Section 1 and EO Section 2

(1) (SF1.2) For each of the following, draw a Venn diagram:
(a) $A \subseteq B, C \subseteq B, A \cap C = \emptyset$

Be sure to draw these diagrams in “most general form.” In (b), for example, B and C are
disjoint, but A and B should show no special relationship.

(b) $A \supseteq C, B \cap C = \emptyset$

(2) (SF 1.3) Let $A = \{w, x, y, z\}$ and $B = \{a, b\}$. List all the elements of the following sets.
(a) $A \times B$
In lexicographic order, the elements of $A \times B$ are

$$(w, a), (w, b), (x, a), (x, b), (y, a), (y, b), (z, a), (z, b).$$

This is the order that the words $wa, wb, xa, xb, etc., would appear in Webster’s Baby
Talk Dictionary. The set $A \times B = \{(w, a), (w, b), (x, a), (x, b), (y, a), (y, b), (z, a), (z, b)\}$,
when written in braces, does not formally imply any particular ordering of the el-
ements. Generally though, any representation of a set will utilize some implicit
ordering of the elements as a part of the “data structure” used.
(b) $B \times A$

In lexicographic order (called lex order for short), the elements of $B \times A$ are

$$(a, w), (a, x), (a, y), (a, z), (b, w), (b, x), (b, y), (b, z).$$

Note that these are different from the elements in the previous part because elements of the Cartesian product are ordered pairs.

(c) $A \times A$

$$(w, w), (w, x), (w, y), (w, z), (x, w), (x, x), (x, y), (x, z), (y, w), (y, y), (y, z),
(z, w), (z, x), (z, y), (z, z)$$

(d) $B \times B$

$$(a, a), (a, b), (b, a), (b, b)$$

(3) (SF 1.6) Each of the following statements about subsets of a set $U$ is false. Draw a Venn diagram to represent the situation being described. In each case, show that the assertion is false by giving counterexamples of the sets.

We omit the Venn diagram. There can be more than one example in each case, so your examples may not be the same as the ones given here.

(a) For all $A$, $B$, and $C$ if $A \not\subseteq B$ and $B \not\subseteq C$ then $A \not\subseteq C$.

A counterexample would be a choice of $A, B, C$ where

$$A \not\subseteq B \land B \not\subseteq C \land A \subseteq C.$$  

We can take $A$ and $B$ to be nonempty disjoint sets and take $C = A$.

(b) For all sets $A$, $B$, and $C$, $(A \cup B) \cap C = A \cup (B \cap C)$.

Take $C = \emptyset$ and $A \neq \emptyset$.

(c) For all sets $A$, $B$, and $C$, $(A - B) \cap (C - B) = A - (B \cup C)$.

Take $A = C \neq \emptyset$ and take $B = \emptyset$. The left hand side is $A$, the right hand side is $\emptyset$.

(d) For all $A$, $B$ and $C$, if $A \cap C \subseteq B \cup C$ and $A \cup C \subseteq B \cup C$ then $A = B$.

A counterexample would be a choice of $A, B, C$ where

$$A \cap C \subseteq B \land A \cup C \subseteq B \cup C \land A \neq B.$$  

Take $C = \emptyset$ and take $A$ to be a proper subset of $B$; that is, $A \subseteq B$, but $A \neq B$.

(e) For all $A$, $B$ and $C$, if $A \cup C = B \cup C$ then $A = B$.

A counterexample would be a choice of $A, B, C$ where

$$A \cup C = B \cup C \land A \neq B.$$  

Take $C$ to be nonempty, $A = C, B = \emptyset$. 
(f) For all sets \( A, B, \) and \( C, (A - B) - C = A - (B - C) \).

Take \( A = B = C \neq \emptyset \). The \( A - (B - C) = A, (A - B) - C = \emptyset - C = \emptyset \).

(4) (SF 1.8) Prove using the definition of set equality, that for all sets \( A, B, \) and \( C, (A - B) \cap (C - B) \subseteq (A \cap C) - B \) and then we will show that \((A - B) \cap (C - B) \supseteq (A \cap C) - B\).

- Let \( x \in U \). If \( x \in (A - B) \cap (C - B) \), then \( x \in A - B \) and hence \( x \in A \). Also, \( x \in C - B \) and hence \( x \in C \). Thus, \( x \in A \cap C \). Also, \( x \notin B \) because \( x \in A - B \).
- Since \( x \in A \cap C \) and \( x \notin B \), \( x \in (A \cap C) - B \). Thus \((A - B) \cap (C - B) \subseteq (A \cap C) - B\).
- Again, let \( x \in U \) and now we work to prove the reverse subset containment. Suppose \( x \in (A \cap C) - B \). Then \( x \in A \) and \( x \in C \) but \( x \notin B \). Thus \( x \in A - B \) and \( x \in C - B \), and so \( x \in (A - B) \cap (C - B) \). Thus \((A \cap C) - B \subseteq (A - B) \cap (C - B)\).

Thus, \((A - B) \cap (C - B) = (A \cap C) - B\).

(5) (EO 2.1) In each case, a binary relation \( R \) on a set \( S \) is specified directly as a subset of \( S \times S \). Determine, for each property, whether the relation \( R \) is reflexive, symmetric, or transitive. Explain your answers.

(a) \( R = \{(0,0), (0,1), (0,3), (1,0), (1,1), (2,3), (3,3)\} \) where \( S = \{0,1,2,3\} \).
   This relation is neither reflexive (2R2), symmetric (0R3 and 3R0), nor transitive (1R0, 0R3 and 1R3).

(b) \( R = \{(1,3), (3,1), (0,3), (3,0), (3,3)\} \) where \( S = \{0,1,2,3\} \).
   This relation is symmetric, but not reflexive and not transitive.

(c) \( R = \{(a,a), (a,b), (b,c), (a,c)\} \) where \( S = \{a, b, c\} \).
   This relation is transitive, but not reflexive and not symmetric.

(d) \( R = \{(a,a), (b,b)\} \) where \( S = \{a, b, c\} \).
   This relation is transitive and symmetric, but not reflexive.

(e) \( R = \emptyset \) where \( S = \{a\} \).
   This relation is transitive and symmetric, but not reflexive.

(6) (EO 2.4) Let \( S = \mathbb{R} \), the real numbers. Define a binary relation on \( S \) by \( xRy \) if \( x^2 = y^2 \). Determine, for each property, whether the relation \( R \) is reflexive, symmetric, or transitive. Explain your answers.

It is reflexive, symmetric and transitive.

- To prove it’s reflexive: let \( x \in \mathbb{R} \). Then \( x^2 = x^2 \) so that \( xRx \).
- To prove it’s symmetric: let \( x, y \in \mathbb{R} \) and suppose \( xRy \). By definition, this means that \( x^2 = y^2 \). By properties of \( = \), we have \( y^2 = x^2 \). So, \( yRx \).
- To prove it’s transitive: let \( x, y, z \in \mathbb{R} \) and suppose that \( xRy \) and \( yRz \). By definition, this means \( x^2 = y^2 \) and \( y^2 = z^2 \). By properties of \( = \), we have \( x^2 = z^2 \) so that \( xRz \).