MATH CSE20 Homework 7
Due Monday November 18

Assigned reading: SF Sections 1 and EO Section 2

(1) (SF 1.2) For each of the following, draw a Venn diagram:
   (a) $A \subseteq B, C \subseteq B, A \cap C = \emptyset$.
   (b) $A \supseteq C, B \cap C = \emptyset$.

(2) (SF 1.3) Let $A = \{w, x, y, z\}$ and $B = \{a, b\}$. List all the elements of the following sets.
   (a) $A \times B$
   (b) $B \times A$
   (c) $A \times A$
   (d) $B \times B$

(3) (SF 1.6) Each of the following statements about subsets of a set $U$ is false. Draw a Venn diagram to represent the situation being described. In each case, show that the assertion is false by giving counterexamples of the sets.
   (a) For all $A, B$ and $C$, if $A \not\subseteq B$ and $B \not\subseteq C$ then $A \not\subseteq C$.
   (b) For all sets $A, B$, and $C$, $(A \cup B) \cap C = A \cup (B \cap C)$.
   (c) For all sets $A, B$, and $C$, $(A - B) \cap (C - B) = A - (B \cup C)$.
   (d) For all $A, B$ and $C$, if $A \cap C \subseteq B \cap C$ and $A \cup C \subseteq B \cup C$ then $A = B$.
   (e) For all $A, B$ and $C$, if $A \cup C = B \cup C$ then $A = B$.
   (f) For all sets $A, B$, and $C$, $(A - B) - C = A - (B - C)$.

(4) (SF 1.8) Prove, using the definition of set equality, that for all sets $A, B$, and $C$,
   $$(A - B) \cap (C - B) = (A \cap C) - B.$$  

(5) (EO 2.1) In each case, a binary relation $R$ on a set $S$ is specified directly as a subset of $S \times S$. Determine, for each property, whether the relation $R$ is reflexive, symmetric, or transitive. Explain your answers.
   (a) $R = \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 3), (3, 3)\}$ where $S = \{0, 1, 2, 3\}$.
   (b) $R = \{(1, 3), (3, 1), (0, 3), (3, 0), (3, 3)\}$ where $S = \{0, 1, 2, 3\}$.
   (c) $R = \{(a, a), (a, b), (b, c), (a, c)\}$ where $S = \{a, b, c\}$
   (d) $R = \{(a, a), (b, b)\}$ where $S = \{a, b, c\}$
   (e) $R = \emptyset$ where $S = \{a\}$.

(6) (EO 2.4) Let $S = \mathbb{R}$, the real numbers. Define a binary relation on $S$ by $xRy$ if $x^2 = y^2$. Determine, for each property, whether the relation $R$ is reflexive, symmetric, or transitive. Explain your answers.