MATH CSE20 Test 2 Review Sheet

Test Tuesday October 29 in lecture: CENTER 115, 3:30pm

Textbook sections: Unit Lo Sections 1 and 2

(1) All questions from Homeworks 3 and 4.

(2) (Lo Review Question 4) The statement form \((p \leftrightarrow r) \rightarrow (q \leftrightarrow r)\) is equivalent to
(a) \([\sim p \vee r] \wedge (p \vee \sim r)] \vee [(\sim q \vee r) \wedge q \vee \sim r]\]
(b) \(\sim [((\sim p \vee r) \wedge (p \vee \sim r)] \wedge [(\sim q \vee r) \wedge (q \vee \sim r)]\]
(c) \([((\sim p \vee r) \wedge (p \vee \sim r)] \wedge [(\sim q \vee r) \wedge (q \vee \sim r)]\]
(d) \(\sim [((\sim p \vee r) \wedge (p \vee \sim r)] \vee [(\sim q \vee r) \wedge (q \vee \sim r)]\]
(e) \(\sim [((\sim p \vee r) \wedge (p \vee \sim r)] \vee [(\sim q \vee r) \wedge (q \vee \sim r)]\]

(3) (Epp 1.2.20) Write negations for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate).
(a) If \(P\) is a square, then \(P\) is a rectangle.
(b) If today is New Year’s Eve, then tomorrow is January.
(c) If the decimal expansion of \(r\) is terminating, then \(r\) is rational.
(d) If \(n\) is prime, then \(n\) is odd or \(n\) is 2.
(e) If \(x\) is nonnegative, then \(x\) is positive or \(x\) is zero.
(f) If Todd is Anna’s father, then Jim is her uncle and Sue is her aunt.
(g) If \(n\) is divisible by 6, then \(n\) is divisible by 2 and \(n\) is divisible by 3.

Now, write the contrapositives, converses, and inverses for each of these statements.

(4) (Epp 1.2.21) Suppose that \(p\) and \(q\) are statements such that \(p \rightarrow q\) is false. Find the truth values of each of the following:
(a) \(\sim p \rightarrow q\)
(b) \(p \vee q\)
(c) \(q \rightarrow p\).

(5) (Epp 2.3.9) Let \(D = E = \{-2, -1, 0, 1, 2\}\). Explain why the following statements are true.
(a) \(\forall x \in D, \exists y \in E, x + y = 0\).
(b) \(\exists x \in D, \forall y \in E, x + y = y\).

(6) (Spring 2013, Midterm 1) Consider the domain of real numbers. Define the predicate \(P(x)\) to mean \(x\) is (strictly) positive, \(N(x)\) to mean \(x\) is (strictly) negative.

A. \(\forall x (P(x) \vee N(x))\).
B. \(\exists x (\sim P(x) \wedge \sim N(x))\).
C. \(\forall x (P(x) \rightarrow \sim N(x))\).
D. \(\exists x (P(x) \rightarrow \sim N(x))\).

(a) Which of the statements above is true? (List the letters.)
(b) Which of the statements can be translated as

Every positive number is not negative.
(c) Which pair of statements are negations of one another? (Give the two letters.)
(d) Give a counterexample which proves that
\[ \forall x(P(x) \rightarrow N(x)) \]
is false. Justify your answer briefly.

(7) (Spring 2013 Midterm 1) Consider a proposition of the form
\[ p \rightarrow (r \lor s). \]
Answer True or False.
(a) The proposition asserts that \( p \) is necessary for \( r \lor s \).
(b) The proposition is equivalent to \( \sim p \lor r \lor s \).
(c) In a direct proof, we may assume that \( r \) and \( s \) are both false.

(8) (Spring 2013 Midterm 1) Fill in the blanks in the following contrapositive proof.

*Any step that relies on something other than basic algebra must be justified/proved as part of your proof.*

- **Thm.** If \( \sqrt{2} \) is not a rational number, then \( 1 + \sqrt{2} \) is not a rational number either.
- **Proof:**
  - Assume (by contrapositive) _________
  - WTS (for contrapositive) _________
    
    *Fill in the body of this proof . . .*
    
  - Conclusion, therefore, _________ QED.

(9) Let \( m, n \) be fixed integers. Prove that if \( m, n \) are both even then \( m + n \) is even. And, prove that if \( m, n \) are both odd then \( m + n \) is even.

(10) Determine whether the property is true for all integers, true for no integers, or true for some integers and false for other integers. Justify your answers.
(a) \( (a + b)^2 = a^2 + b^2 \)
(b) The average of any two odd integers is odd.

(11) Find the truth set of each predicate.
(a) Predicate: \( \frac{6}{3} \) is an integer; Domain: \( \mathbb{Z} \)
(b) Predicate: \( \frac{6}{3} \) is an integer; Domain: \( \mathbb{N} \)
(c) Predicate: \( 1 \leq x^2 \leq 4 \); Domain: \( \mathbb{R} \)
(d) Predicate: \( 1 \leq x^2 \leq 4 \); Domain: \( \mathbb{Z} \)