Problem 1

a. Show that, if a DFA \( M = (\Sigma, Q, q_0, F, \delta) \) accepts a string \( w \in \Sigma^* \) with \( |w| \geq |Q| \), then it also accepts some string \( y \in \Sigma^* \) with \( |y| < |Q| \).

b. Use the fact in part a. to give an algorithm for deciding \( E_{DFA} \), the emptiness problem for DFAs, that is different than the one we gave in class.

Problem 2

Let \( L_2 \) be the language

\[
\{ \langle G \rangle \mid G \text{ is a CFG and } L(G) \text{ includes only finitely many strings} \}.
\]

Is \( L_2 \) decidable (i.e., recursive)? If so, describe a Turing machine that decides \( L_2 \); if not, prove that \( L_2 \) is undecidable.

Problem 3

Let \( L_3 \) be the language

\[
\{ \langle M, w \rangle \mid M \text{ is a Turing machine, } w \text{ is a string, and } M \text{ never moves its head left when run on input } w. \}
\]

Is \( L_3 \) decidable (i.e., recursive)? If so, describe a Turing machine that decides \( L_3 \); if not, prove that \( L_3 \) is undecidable.

Problem 4

Let \( L_4 \) be the language

\[
\{ \langle M, w, q \rangle \mid M \text{ is a Turing machine, } w \text{ is a string, and } q \text{ is a state;} \}
\text{ and } M, \text{ when run on input } w, \text{ never enters the state } q. \}
\]

Is \( L_4 \) decidable (i.e., recursive)? If so, describe a Turing machine that decides \( L_4 \); if not, prove that \( L_4 \) is undecidable.

Problem 5

Closure properties:

a. Show that the class of R.E. languages is not closed under complement.

b. Show that the class of decidable languages is closed under union.