Problem 1 Warmup.

- a. Prove that if a language $L$ contains finitely many elements then it is regular.
- b. If a language $L$ contains infinitely many elements, can it still be regular?
- c. Give the state diagram for a DFA accepting the language consisting of all strings over $\Sigma = \{0, 1\}$ that do not end with the string “0101”. For example, the string “010” is in the language, whereas “000101” is not.

Problem 2 We defined DFAs to have a set $F$ of accepting states. In this problem we consider a variant of DFAs that have exactly one accepting state, $q_{\text{accept}}$. We call this variant of DFAs one-DFAs.

- a. Give a formal definition of one-DFAs, including the parts of a one-DFA, what it means for a one-DFA to accept a string $w$, and the language of a one-DFA.
- b. Show that for any one-DFA $M_1$ there exists an equivalent DFA $M$ (i.e., one such that $L(M_1) = L(M)$).
- c. Show that the converse is not true: there exist regular languages (i.e., languages recognized by some DFA) that are not recognized by any one-DFA.
- d. Consider an analogous definition of one-NFAs, NFAs with exactly one accepting state. How do one-NFAs compare to NFAs?

Problem 3 We say that a language is 2-regular if it is recognized by some DFA that has at most 2 states.

- a. Show that the class of 2-regular languages is closed under complement.
- b. Show that the class of 2-regular languages is not closed under union.
- c. Show that the class of 2-regular languages is not closed under intersection.

**Hint:** For parts b. and c., Try enumerating all 2-regular languages over a unary alphabet. De Morgan’s laws may also save you some work.