Radiometry and Reflectance

Announcements

- Assignment 1 Posted to the web page
- Read Chapter 4 of Forsyth & Ponce
- Textbook warning: Last year, some students have used an online version of the text. I believe that it’s a draft version and it doesn’t include chapters 2 & 3

Appearance: lighting, surface reflectance

A local coordinate system on a surface

- Consider a point \( P \) on the surface
- Light arrives at \( P \) from a hemisphere of directions defined by the surface normal \( N \)
- We can define a local coordinate system whose origin is \( P \) and with one axis aligned with \( N \)
- Convenient to represent in spherical angles.

Foreshortening

Measuring Angle

- The solid angle subtended by an object from a point \( P \) is the area of the projection of the object onto the unit sphere centered at \( P \).
- Definition is analogous to projected angle in 2D
- Measured in steradians, sr
- If I’m at \( P \), and I look out, solid angle tells me how much of my view is filled with an object

- The surface is foreshortened by the cosine of the angle between the normal and direction to the light.
Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point.
- The solid angle subtended by a patch area $dA$ is given by:

$$d\omega = \frac{dA \cos \theta}{r^2}$$

Radiance

- Power is energy per unit time.
- Radiance: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle.
- Symbol: $L(x, \theta, \phi)$
- Units: watts per square meter per steradian: $\text{w/(m}^2\text{sr})$

Irradiance

- How much light is arriving at a surface?
- Units of irradiance: Watts/m²
- This is a function of incoming angle.
- A surface experiencing radiance $L(x, \theta, \phi)$ coming in from solid angle $d\omega$ experiences irradiance $E(x)$:

$$E(x) = L(x, \theta, \phi) \cos \theta d\omega$$

Radiometry of thin lenses

- What is image irradiance $E$ for a radiance $L$ emitted from a point $P$?

Solution is a substantial example problem.
A small patch on the surface $\delta A$ will project to a small patch on the image $\delta A'$. Let $\delta \omega$ be the solid angle subtended by $\delta A$ (or $\delta A'$) from the center of the lens.

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The power $\delta P$ emitted from the patch $\delta A$ with radiance $L$ and falling on the lens is:

$$\delta P = \frac{\pi}{4} \frac{d}{z'^2} \cos^4 \alpha L$$

where:
- $E$: Image irradiance
- $L$: emitted radiance
- $d$: Lens diameter
- $z'$: depth of image plane
- $\alpha$: Angle of patch from optical axis

**Camera’s sensor**

- Measured pixel intensity is a function of irradiance integrated over:
  - pixel’s area
  - over a range of wavelengths
  - for some period of time

$$I = \iiint E(x, y, \lambda, t) s(x, y) q(\lambda) dy dx d\lambda dt$$

- Ideally, it’s linear to the radiance, but the camera response $C(.)$ may not be linear

$$I = C \left( \iiint E(x, y, \lambda, t) s(x, y) q(\lambda) dy dx d\lambda dt \right)$$
Color Cameras

We considered four concepts:
1. Prism (with 3 sensors)
2. Filter mosaic
3. Filter wheel
4. Fovean

Light at surfaces

Many effects when light strikes a surface -- could be:
- transmitted
  - Skin, glass
- reflected
  - mirror
- scattered
  - milk
- travel along the surface and leave at some other point
- absorbed
  - sweaty skin

Assume that:
- surfaces don’t fluoresce -- e.g. scorpions, detergents
- surfaces don’t emit light (i.e. are cool)
- all the light leaving a point is due to that arriving at that point

BRDF

With assumptions in previous slide:
- Bi-directional Reflectance Distribution Function
  \[ \rho(\theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) \]
- Ratio of emitted radiance to incident irradiance (units: sr^-1)
- Function of:
  - Incoming light direction: \( \theta_{\text{in}}, \phi_{\text{in}} \)
  - Outgoing light direction: \( \theta_{\text{out}}, \phi_{\text{out}} \)

\[ \rho(\theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) = \frac{L_o(x, \theta_{\text{out}}, \phi_{\text{out}})}{L_i(x, \theta_{\text{in}}, \phi_{\text{in}}) \cos \theta_{\text{in}} d\omega} \]

Where \( \rho \) is sometimes denoted \( f_r \).

The Reflection Equation

\[ L_r(x, \omega_i) = \int_{\omega} f_r(x, \omega_i \rightarrow \omega_j) L_i(x, \omega_i) \cos \theta_i d\omega_i \]

where \( \omega_j = (\theta, \phi) \)

Properties of BRDFs

1. Non-negative: \( \rho(\theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) \geq 0 \)
2. Helmholtz Reciprocity Principle:
   \[ \rho(\theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) = \rho(\theta_{\text{out}}, \phi_{\text{out}}; \theta_{\text{in}}, \phi_{\text{in}}) \]
3. Total energy leaving a surface must be less than total energy arriving at the surface

Surface Reflectance Models

Common Models
- Lambertian
- Phong
- Physics-based
  - Specular
    - Blinn [1977], Cook-Torrance [1982], Ward [1992]
  - Diffuse
    - Hanrahan, Kreuger [1993]
  - Generalized Lambertian
    - Oren, Nayar [1995]
  - Thoroughly Pitted Surfaces
    - Koenderink et al [1999]
- Phenomenological
  - Koenderink, Van Doorn [1996]

Arbitrary Reflectance

- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement

Specialized
- Hair, skin, threads, paper

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Computer Vision I
Isotropic BRDF's are symmetric about the surface normal. If the surface is rotated about the normal for the same incident and emitting directions, the value of the BRDF is the same.

Isotropic BRDF

\[ f(\theta_0, \varphi_0, \theta_i, \varphi_i) = f(\theta_0, \theta_i; \varphi_i - \varphi_0) \]

Lambertian (Diffuse) Surface

- BRDF is a constant called the albedo. \( \rho(\omega_i, \omega_r, \theta, \varphi) = K \)
- Emitted radiance is NOT a function of outgoing direction – i.e. constant in all directions.
- For lighting coming in single direction \( \omega_i \), emitted radiance is proportional to cosine of the angle between normal and light direction

\[ L_r = KN \cdot \omega_i \]

Specular Reflection: Smooth Surface

Rough Specular Surface
Non-Lambertian reflectance

Ways to measure BRDF’s