CS 140 Lecture 6: Other Types of Gates

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Combinational Logic: Other Types of Gates

- Universal Set of Gates
- Other Types of Gates
  1) XOR
  2) NAND / NOR
  3) Block Diagram Transfers
Universal Set

Universal Set: A set of gates such that every switching function can be implemented with gates in this set.

Ex:

{AND, OR, NOT}

{AND, NOT}

{OR, NOT}
Universal Set

Universal Set: A set of gates such that every Boolean function can be implemented with gates in this set.

Ex:

\{\text{AND, OR, NOT}\}

\{\text{AND, NOT}\} \ OR \text{ can be implemented with AND \& NOT gates} \quad a+b = (a'b')'

\{\text{OR, NOT}\} \ AND \text{ can be implemented with OR \& NOT gates} \quad ab = (a'+b')'

\{\text{XOR}\} \text{ is not universal}

\{\text{XOR, AND}\} \text{ is universal}
Is the set \{AND, OR\} (but no NOT gate) universal?

A. Yes

B. No
X \oplus 1 = X\cdot1' + X'\cdot1 = X' \text{ if constant "1" is available.}
Universal Set

Remark: Universal set is a powerful concept to identify the coverage of a set of gates afforded by a given technology.
Other Types of Gates

1) XOR \( X \oplus Y = XY' + X'Y \)

(a) Commutative \( X \oplus Y = Y \oplus X \)
(b) Associative \( (X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) \)
(c) \( 1 \oplus X = X' \quad 0 \oplus X = 0X' + 0'X = X \)
(d) \( X \oplus X = 0, \quad X \oplus X' = 1 \)
e) if \( ab = 0 \) then \( a \oplus b = a + b \)

**Proof:** If \( ab = 0 \) then  
\[
\begin{align*}
    a &= a \cdot (b + b') = ab + ab' = ab' \\
    b &= b \cdot (a + a') = ba + ba' = a'b \\
    a + b &= ab' + a'b = a \oplus b
\end{align*}
\]

f) \( X \oplus X Y' \oplus X' Y \oplus (X + Y) \oplus X = ?? \)

To answer, we apply Shannon’s Expansion.
Shannon’s Expansion (for switching functions)

Formula: \( f(x, Y) = x \cdot f(1, Y) + x' \cdot f(0, Y) \)

Proof by enumeration:
If \( x = 1 \), \( f(x, Y) = f(1, Y) : \) \( 1 \cdot f(1, Y) + 1' \cdot f(0, Y) = f(1, Y) \)

If \( x = 0 \), \( f(x, Y) = f(0, Y) : \) \( 0 \cdot f(1, Y) + 0' \cdot f(0, Y) = f(0, Y) \)
Back to our problem…

\[ X \oplus XY' \oplus X'Y \oplus (X + Y) \oplus X = ? \]

\[ \Rightarrow \quad X \oplus (XY') \oplus (X'Y) \oplus (X + Y) \oplus X = f (X, Y) \]

If \( X = 1 \), \( f (1, Y) = 1 \oplus Y' \oplus 0 \oplus 1 \oplus 1 = Y \)
If \( X = 0 \), \( f (0, Y) = 0 \oplus 0 \oplus Y \oplus Y \oplus 0 = 0 \)

Thus, \( f (X, Y) = XY \)
XOR gates

iClicker: \( a + (b \oplus c) = (a+b) \oplus (a+c) \) ?
A. Yes
B. No
2) NAND, NOR gates

NAND, NOR gates are not associative

Let \( a \mid b = (ab)' \)

\[(a \mid b) \mid c \neq a \mid (b \mid c)\]
3) Block Diagram Transformation

a) Reduce # of inputs.
b. DeMorgan’s Law

\[(a+b)' = a'b'\]

\[(ab)' = a'+b'\]
c. Sum of Products (Using only NAND gates)

Sum of Products (Using only NOR gates)
d. Product of Sums (NOR gates only)
NAND, NOR gates

Remark:
Two level NAND gates: Sum of Products
Two level NOR gates: Product of Sums
Part II. Sequential Networks

Memory / Timesteps

Clock

Flip flops
Specification
Implementation