Problem 1 Let $L_1$ be the language

$$\{ \langle G \rangle \mid G \text{ is a CFG and } L(G) \text{ includes only finitely many strings}\}.$$ 

Is $L_1$ decidable (i.e., recursive)? If so, describe a Turing machine that decides $L_1$; if not, prove that $L_1$ is undecidable.

Problem 2 $L_2$ be the language

$$\{ \langle M, w, q \rangle \mid M \text{ is a Turing machine, } w \text{ is a string, and } q \text{ is a state;} \}
\text{ and } M, \text{ when run on input } w, \text{ never enters the state } q.$$}

Is $L_2$ decidable (i.e., recursive)? If so, describe a Turing machine that decides $L_2$; if not, prove that $L_2$ is undecidable.

Problem 3 Let $L_3$ be the language

$$\{ \langle M, w \rangle \mid M \text{ is a Turing machine, } w \text{ is a string, and } M \text{ does not visit any state more than once when run on input } w. \}.$$ 

Is $L_3$ decidable (i.e., recursive)? If so, describe a Turing machine that decides $L_3$; if not, prove that $L_3$ is undecidable.

Problem 4 Closure properties:

a. Show that the class of R.E. languages is not closed under complement.

b. Show that the class of decidable languages is closed under concatenation.