Stereo

Computer Vision I
CSE252A
Lecture 14

Binocular Stereopsis: Mars
Given two images of a scene where relative locations of cameras are known, estimate depth of all common scene points.

Two images of Mars

An Application: Mobile Robot Navigation
The INRIA Mobile Robot, 1990.

Mars Exploratory Rovers: Spirit and Opportunity

Commercial Stereo Heads
Trinocular stereo Binocular stereo

Stereo can work well
Stereo Vision Outline

- Offline: Calibrate cameras & determine “epipolar geometry”
- Online
  1. Acquire stereo images
  2. Rectify images to convenient epipolar geometry
  3. Establish correspondence
  4. Estimate depth

Reconstruction: General 3-D case

Given two image measurements \( p \) and \( p' \), estimate \( P \).

- Linear Method: find \( P \) such that
  \[
  \begin{align*}
  p \times MP &= 0 \\
  p' \times MP' &= 0
  \end{align*}
  \iff
  \[
  \begin{bmatrix}
  p_1 & M \end{bmatrix} P = 0
  \]

- Non-Linear Method: find \( Q \) minimizing
  \[
  d^2(p, q) + d^2(p', q')
  \]
  where \( q^r = MQ \) and \( q'^r = M'Q' \)

Two Approaches

1. Feature-Based
   - From each image, process “monocular” image to obtain cues (e.g., corners, lines).
   - Establish correspondence between

2. Area-Based
   - Directly compare image regions between the two images.
Human Stereopsis: Binocular Fusion

How are the correspondences established?

Julesz (1971): Is the mechanism for binocular fusion a monocular process or a binocular one??
- There is anecdotal evidence for the latter (camouflage).
- Random dot stereograms provide an objective answer

Random Dot Stereograms

• Detail of a 1639 etching.

• In Rembrandt's painted self-portraits (left panel) in which the eyes are clearly visible, his left eye frequently looks straight out and the right off to the side. It is the opposite in his etchings (right panel).

Need for correspondence
Epipolar Constraint

- Potential matches for \( p \) have to lie on the corresponding epipolar line \( l' \).
- Potential matches for \( p' \) have to lie on the corresponding epipolar line \( l \).

Epipolar Geometry

- Epipolar Plane
- Epipoles
- Epipolar Lines
- Baseline

Family of epipolar Planes

Family of planes \( \pi \) and lines \( l \) and \( l' \) intersect in \( e \) and \( e' \)

Skew Symmetric Matrix & Cross Product

- The cross product \( a \times b \) of two vectors \( a \) and \( b \) can be expressed as a matrix vector product \( [a]b \) where \( [a] \) is the skew symmetric matrix:

\[
[a] = \begin{bmatrix}
0 & -a_3 & a_2 \\
a_3 & 0 & -a_1 \\
-a_2 & a_1 & 0
\end{bmatrix}
\]

- A matrix \( S \) is skew symmetric iff \( S = -S^T \)

Epipolar Constraint: Calibrated Case

\[
\partial_p \cdot (\partial p \times \partial p') = 0 \quad p \cdot [t \times (Rp')] = 0 \quad \text{with} \quad \begin{cases}
p = (u, v, 1)^T \\
p' = (u', v', 1)^T \\
M = \begin{bmatrix} I_d & 0 \end{bmatrix} \\
M' = \begin{bmatrix} R^T & -R^T t \end{bmatrix}
\end{cases}
\]

Calibration

Determine intrinsic parameters and extrinsic relation of two cameras
The Eight-Point Algorithm (Longuet-Higgins, 1981)
Much more on multi-view in CSE252B

\[
\begin{pmatrix}
F_{11} & F_{12} & F_{13} \\
F_{21} & F_{22} & F_{23} \\
F_{31} & F_{32} & F_{33}
\end{pmatrix}
\begin{pmatrix}
u' \\ v' \\ 1
\end{pmatrix} = 0
\Rightarrow
\begin{pmatrix}
\sum (p_i^T F p_i')^2 \\
\sum (p_i^T F p_i)^2
\end{pmatrix} = 0
\]

Set \( F_{33} \) to 1

\[
\begin{pmatrix}
F_{11} & F_{12} & F_{13} \\
F_{21} & F_{22} & F_{23} \\
F_{31} & F_{32} & F_{33}
\end{pmatrix}
\begin{pmatrix}
u' \\ v' \\ 1
\end{pmatrix} = 0
\]

Minimize:

\[
\sum (p_i^T F p_i')^2
\]

under the constraint

\[
\sum (p_i^T F p_i)^2 = 1.
\]

Properties of the Essential Matrix

\( E p' = 0 \) with \( E - [t]_R \)

- \( E p' \) is the epipolar line associated with \( p' \).
- \( E^T p \) is the epipolar line associated with \( p \).
- \( E e' = 0 \) and \( E^T e = 0 \).
- \( E \) is singular.
- \( E \) has two equal non-zero singular values (Huang and Faugeras, 1989).

Example: converging cameras
courtesy of Andrew Zisserman

Example: motion parallel with image plane
courtesy of Andrew Zisserman (simple for stereo -> rectification)

Example: forward motion
courtesy of Andrew Zisserman
Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.

Image pair rectification
simplify stereo matching by warping the images

Apply projective transformation $H$ so that epipolar lines correspond to horizontal scanlines

$H e = e$

map epipole $e$ to $(1,0,0)$
try to minimize image distortion
Note that rectified images usually not rectangular

Rectified Images
See Section 7.3.7 for specific method

Multiple Interpretations
Each feature on left epipolar line match one and only one feature on right epipolar line.
Each feature on left epipolar line match one and only one feature on right epipolar line.