Problem 1 (6 points)

Write each of the following statements in symbolic form using the symbols $\neg$, $\land$, $\lor$ and $\rightarrow$, and atomic propositions $P$="It is a duck", $Q$ = “It talks like a duck”, $R$="It walks like a duck".

1. If it walks like a duck and it talks like a duck, then it is a duck.
2. Either it does not walk like a duck or it does not talk like a duck, or or it is not a duck.
3. If it does not walk like a duck or it does not talk like a duck, then it is not a duck.

Use the truth table method to determine if any of the 3 statements above are logically equivalent. More specifically, draw a single truth table for all three statements, and asserts which pairs of statements are logically equivalent or not. [As a reminder, the truth table should contain a column for each of the statements (1), (2) and (3) in symbolic form, and each of their subexpressions, including the atomic statements $P$, $Q$ and $R$.]

Problem 2 (10 points)

The NAND logical connective $P \uparrow Q$ is defined by the logical equivalence $P \uparrow Q = \neg(P \land Q)$, and it is important because it is easy to build an electronic circuit that computes the NAND of two signals. In this problem you will show that the NAND connective is complete, in the sense that any other logic operation can be expressed in terms of NAND alone. Below, by “NAND expression” we mean a logical statement that makes use of NAND as its only logical connective.

1. Give a NAND expression for $P \land Q$, and prove your answer correct using the truth table method
2. Give a NAND expression for $P \lor Q$, and prove your answer correct using the truth table method
3. Give a NAND expression for $\neg P$, and prove your answer correct using the truth table method.
4. Give a NAND expression for the constant $T$, and prove your answer correct using the truth table method.
5. Give a NAND expression for the constant $F$, and prove your answer correct using the truth table method.

[If you need help, you can find expressions for the first two questions in the Exercise 1.1 section of the textbook.]

Problem 3 [6 points]

Use the truth table method to prove that the logical and operation is associative, i.e., $(P \land Q) \land R$ is logically equivalent to $P \land (Q \land R)$.

Then, answer the following question: is the same true for the NAND operation? I.e., is the expression $(P \uparrow Q) \uparrow R$ is logically equivalent to $P \uparrow (Q \uparrow R)$? Prove or disprove using the truth table method.