Photometric Stereo

Computer Vision I
CSE252A
Lecture 8

Last lecture in a nutshell

- BRDF
- Light Sources
  - Point source
  - Point at infinity
  - Line & area sources
  - Spherical harmonics
- Shadows
- Inter-reflections

Light Sources

- Point sources
- Distant point sources (point source at infinity)
- Line source
- Area source
- Spherical Harmonics (diffuse, distant lighting)
- Light field (function on 4-D space)

Marschner's Image-Based BRDF Measurement

- For uniform BRDF, capture 2-D slice corresponding to variations in normals

Diffuse lighting at infinity: Spherical Harmonics

\( Y_{lm}(\theta, \varphi) \)
Shadows cast by a point source

- A point that can’t see the source is in shadow
- For point sources, the geometry is simple

Shading models

Local shading model
- Surface has incident radiance due only to sources visible at each point
- Advantages:
  - Often easy to manipulate, expressions easy
  - Supports quite simple theories of how shape information can be extracted from shading
- Used in vision & real-time graphics

Global shading model
- Surface radiosity is due to radiance reflected from other surfaces as well as from surfaces
- Advantages:
  - Usually very accurate
- Disadvantage:
  - Extremely difficult to infer anything from shading values
- Rarely used in vision, often in photorealistic graphics

Photometric Stereo

Photometric Stereo Rigs

Photometric Stereo: General BRDF and Reflectance Map
Coordinate system

Surface: $s(x,y) = (x,y, f(x,y))$
Tangent vectors:
\[
\frac{ds(s,y)}{dx} = \left(1, 0, \frac{df}{dx}\right)
\]
\[
\frac{ds(x,y)}{dy} = \left(0, 1, \frac{df}{dy}\right)
\]
Normal vector:
\[
\mathbf{n} = \frac{\mathbf{s}_x \times \mathbf{s}_y}{|\mathbf{s}_x \times \mathbf{s}_y|}
\]

Coordinate system

Gradient Space: $(p,q)$

Image Formation

For a given point $A$ on the surface lit by a point source at infinity, the image irradiance $E(x,y)$ is a function of
1. The BRDF at $A$
2. The surface normal at $A$
3. The direction of the light source

Reflectance Map

Viewer center representation of reflectance

Let the BRDF be the same across the surface and let the light source be distance $(s)$ be constant.
1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, this can be expressed as $R(p,q)$.

Example Reflectance Map: Lambertian surface

lit from direction as camera is viewing surface $(p,q)$

$R(p,q)$

Lighting from front
What does the intensity (Irradiance) of one pixel in one image tell us? (e.g., let's say the intensity is 0.3)

Then, the normal lies on this curve

A third image would disambiguates between two possible normals

One viewpoint, two images, two light sources
Two super imposed reflectance maps

Three Source Photometric stereo:
Step 1
Offline:
Using source directions & BRDF, construct reflectance map for each light source direction. $R_1(p,q), R_2(p,q), R_3(p,q)$

Online:
1. Acquire three images with known light source directions. $E_1(x,y), E_2(x,y), E_3(x,y)$
2. For each pixel location $(x,y)$, find $(p,q)$ as the intersection of the three curves $R_1(p,q)=E_1(x,y)$ $R_2(p,q)=E_2(x,y)$ $R_3(p,q)=E_3(x,y)$
3. This is the surface normal at pixel $(x,y)$. Over image, the normal field is estimated.

Normal Field

Plastic Baby Doll: Normal Field
Next step:
Go from normal field to surface

Recovering the surface \( f(x,y) \)

Many methods: Simplest approach
1. From estimate \( n = (n_x, n_y, n_z) \), \( p = n_x/n_z \), \( q = n_y/n_z \)
2. Integrate \( p = df/\partial x \) along a row \((x,0)\) to get \( f(x,0) \)
3. Then integrate \( q = df/\partial y \) along each column starting with value of the first row

What might go wrong?

• Height \( z(x,y) \) is obtained by integration along a curve from \((x_0, y_0)\):
  \[ z(x,y) = z(x_0, y_0) + \int_{(x_0, y_0)}^{(x, y)} (pdx + qdy) \]
• If one integrates the derivative field along any closed curve, one expects to get back to the starting value.
• Might not happen because of noisy estimates of \((p, q)\)

What might go wrong?

Integrability. If \( f(x,y) \) is the height function, we expect that
\[ \frac{\partial f}{\partial y} = \frac{\partial f}{\partial x} \]
In terms of estimated gradient space \((p, q)\), this means:
\[ \frac{\partial p}{\partial y} = \frac{\partial q}{\partial x} \]
But since \( p \) and \( q \) were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold

Horn’s Method

[ “Robot Vision, B.K.P. Horn, 1986” ]

• Formulate estimation of surface height \( z(x,y) \) from gradient field by minimizing cost functional:
  \[ \int_{\text{image}} (z_x - p)^2 + (z_y - q)^2 \, dx \, dy \]
  where \((p, q)\) are estimated components of the gradient while \( z_x \) and \( z_y \) are partial derivatives of best fit surface
• Solved using calculus of variations – iterative updating
• \( z(x,y) \) can be discrete or represented in terms of basis functions.
• Integrability is naturally satisfied.

II. Photometric Stereo:
Lambertian Surface,
Distant Known Lighting
Lambertian Surface

At image location (u,v), the intensity of a pixel \( x(u,v) \) is:

\[
e(u,v) = [a(u,v) \hat{n}(u,v)] \cdot [s_0 s]
= b(u,v) \cdot s
\]

where

- \( a(u,v) \) is the albedo of the surface projecting to (u,v).
- \( \hat{n}(u,v) \) is the direction of the surface normal.
- \( s_0 \) is the light source intensity.
- \( s \) is the direction to the light source.

Lambertian Photometric stereo

- If the light sources \( s_1, s_2, \) and \( s_3 \) are known, then we can recover \( b \) at each pixel from as few as three images. (Photometric Stereo: Silver 80, Woodham81).

\[
[e_1 \ e_2 \ e_3] = b^T [s_1 \ s_2 \ s_3]
\]

- i.e., we measure \( e_1, e_2, \) and \( e_3 \) and we know \( s_1, s_2, \) and \( s_3 \). We can then solve for \( b \) by solving a linear system.

\[
b^T = [e_1 \ e_2 \ e_3]^T [s_1 \ s_2 \ s_3]
\]

- Surface normal is: \( n = b / \|b\| \), albedo is: \( |b| \)

What if we have more than 3 Images?
Linear Least Squares

Let the residual be

\[
\tau = e - Sb
\]

Squaring this:

\[
\tau^T \tau = (e - Sb)^T (e - Sb) = e^T e - 2b^T S^T e + b^T S^T S b
\]

\[
\nabla_{b}(\tau^2) = 0 - \text{zero derivative is a necessary condition for a minimum, or}
\]

\[
-2S^T e + 2S^T S b = 0;
\]

Solving for \( b \) gives

\[
b = (S^T S)^{-1} S^T e
\]

Input Images

Recovered albedo

Recovered normal field
Surface recovered by integration

Lambertian Photometric Stereo

Reconstruction with albedo map

Without the albedo map

Another person

No Albedo map
III. Photometric Stereo with unknown lighting and Lambertian surfaces

Covered in Illumination cone slides