Cameras and Radiometry

Computer Vision I
CSE 252A
Lecture 5

Last lecture in a nutshell

Conversion
Euclidean -> Homogenous -> Euclidean

In 2-D
- Euclidean -> Homogenous: \((x, y) \rightarrow k(x, y, 1)\)
- Homogenous -> Euclidean: \((x, y, 1) \rightarrow \frac{x}{k}, \frac{y}{k}\)

In 3-D
- Euclidean -> Homogenous: \((x, y, z) \rightarrow k(x, y, z, 1)\)
- Homogenous -> Euclidean: \((x, y, z, w) \rightarrow \frac{x}{w}, \frac{y}{w}, \frac{z}{w}\)

Affine Camera Model

- Take perspective projection equation, and perform Taylor series expansion about some point \((x_0, y_0, z_0)\).
- Drop terms that are higher order than linear.
- Resulting expression is affine camera model

Simplified Camera Models

Perspective Projection
Affine Camera Model
Scaled Orthographic Projection
Orthographic Projection

Coordinate Changes: Rigid Transformations

Euclidean

\[
B P = A R A P + B O_A
\]

Homogeneous

\[
[\hat{A} P] = \begin{bmatrix}
\hat{A} R & \hat{A} O_A \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{A} P \\
1
\end{bmatrix}
\]
Some points about SO(n)

- $SO(n) = \{ R \in \mathbb{R}^{nxn} : R^T R = I, \det(R) = 1 \}$
  - $SO(2)$: rotation matrices in plane $\mathbb{R}^2$
  - $SO(3)$: rotation matrices in 3-space $\mathbb{R}^3$
- Forms a Group under matrix product operation:
  - Identity
  - Inverse
  - Associative
  - Closure
- Closed (finite intersection of closed sets)
- Bounded $R_{ij} \in [-1, +1]$
- Does not form a vector space.
- Manifold of dimension $n(n-1)/2$
  - $\text{Dim}(SO(2)) = 1$
  - $\text{Dim}(SO(3)) = 3$

SO(3)

- Parameterizations of SO(3)
  - 3-D manifold, so between 3 parameters and $2n + 1$ parameters (Whitney’s Embedding Thm.)
    - Roll-Pitch-Yaw
    - Euler Angles
    - Axis Angle (Rodrigues formula)
    - Cayley’s formula
    - Matrix Exponential
    - Quaternions (four parameters + one constraint)

Camera parameters

- Issue
  - World units (e.g., cm), camera units (pixels)
  - Camera may not be at the origin, looking down the z-axis
    - Extrinsic parameters
    - One unit in camera coordinates may not be the same as one unit in world coordinates
    - Intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

\[
\begin{pmatrix}
  T \\
  V \\
  W
\end{pmatrix}
= \begin{pmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0
\end{pmatrix} \begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix}
\]

$3 \times 3$ Transformation representing intrinsic parameters

$4 \times 4$: Rigid transformation representing extrinsic parameters

Camera Calibration

- Given $n$ points $P_1, \ldots, P_n$ with known positions and their images $p_1, \ldots, p_n$, estimate intrinsic and extrinsic camera parameters
- See Text book for how to do it.
- Camera Calibration Toolbox for Matlab (Bouguet)
  - http://www.vision.caltech.edu/bouguetj/calib_doc/

Qualcomm Augmented Reality Demo

- http://www.youtube.com/watch?v=hXtq1QJMLiw&feature=player_embedded

Projective Transformation (Homography)

- $3 \times 3$ linear transformation of homogenous coordinates
- Points map to points,
- Lines map to lines

\[
\begin{pmatrix}
  u_1 \\
  u_2 \\
  u_3
\end{pmatrix} =
\begin{pmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{pmatrix}
\begin{pmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{pmatrix}
\]
Image of a plane

- The mapping between coordinates in a plane in 3-D to the image plane under perspective is a projective transformation

\[
\begin{bmatrix}
    u_1 \\
    u_2 \\
    u_3
\end{bmatrix} =
\begin{bmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    1
\end{bmatrix}
\]

- A square maps to an arbitrary quadrilateral
- Under affine or scaled orthographic camera model, square maps to a parallelogram

Beyond the pinhole Camera
Getting more light – Bigger Aperture

Pinhole Camera Images with Variable Aperture

<table>
<thead>
<tr>
<th>Aperture (mm)</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td><img src="image1" alt="Image" /></td>
</tr>
<tr>
<td>.6</td>
<td><img src="image2" alt="Image" /></td>
</tr>
<tr>
<td>.35</td>
<td><img src="image3" alt="Image" /></td>
</tr>
<tr>
<td>.15</td>
<td><img src="image4" alt="Image" /></td>
</tr>
<tr>
<td>.07</td>
<td><img src="image5" alt="Image" /></td>
</tr>
</tbody>
</table>

Limits for pinhole cameras

![Image](image6)

2.18. DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a tiny mirror were made using pinholes of decreasing size. (A) When the pinhole is small, the image is not properly conveged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further ruins the focus, due to diffraction. From Rothwell, 1958.

The reason for lenses

- Rotationally symmetric about optical axis.
- Spherical interfaces.

Thin Lens

- Rotationally symmetric about optical axis.
- Spherical interfaces.
Thin Lens: Center

• All rays that enter lens along line pointing at \( O \) emerge in same direction.

Thin Lens: Focus

Parallel lines pass through the focus, \( F \).

Thin Lens: Image of Point

All rays passing through lens and starting at \( P \) converge upon \( P' \).

Thin Lens: Image Plane

A price: Whereas the image of \( P \) is in focus, the image of \( Q \) isn’t.

Thin Lens: Aperture

• Smaller Aperture -> Less Blur
• Pinhole -> No Blur
Deviations from the lens model

Deviations from this ideal are **aberrations**

Two types

1. geometrical
   - spherical aberration
   - astigmatism
   - distortion
   - coma

2. chromatic
   Aberrations are reduced by combining lenses

**Compound lenses**

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**Spherical aberration**

Rays parallel to the axis do not converge

Outer portions of the lens yield smaller focal lengths

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**Astigmatism**

An optical system with astigmatism is one where rays that propagate in two perpendicular planes have different foci. If an optical system with astigmatism is used to form an image of a cross, the vertical and horizontal lines will be in sharp focus at two different distances.

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**Distortion**

Magnification/focal length different for different angles of inclination

Can be corrected (if parameters are know)

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**Chromatic aberration**

(great for prisms, bad for lenses)
Chromatic aberration

rays of different wavelengths focus in different planes

The image is blurred and appears colored at the fringe.

Chromatic aberration cannot be removed completely

sometimes achromatization is achieved for more than 2 wavelengths

Vignetting: Spatial Non-Uniformity

Vignetting

– Only part of the light reaches the sensor
– Periphery of the image is dimmer

Appearance and surface reflectance

Radiometry

• Read Chapter 4 of Ponce & Forsyth
• Solid Angle
• Irradiance
• Radiance
• BRDF
• Lambertian/Phong BRDF

A local coordinate system on a surface

• Consider a point \( P \) on the surface
• Light arrives at \( P \) from a hemisphere of directions defined by the surface normal \( N \)
• We can define a local coordinate system whose origin is \( P \) and with one axis aligned with \( N \)
• Convenient to represent in spherical angles.
Foreshortening

Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point.

- The solid angle subtended by a patch area $dA$ is given by:

$$d\omega = \frac{dA \cos \theta}{r^2}$$

Radiance

- Power is energy per unit time.

- Radiance: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle.

- Symbol: $L(x, \theta, \phi)$

- Units: watts per square meter per steradian: $w/(m^2sr^1)$

Radiance properties

- In free space, radiance is constant as it propagates along a ray.
  - Derived from conservation of flux.
  - Fundamental in Light Transport.

- Power emitted from patch, but radiance in direction different from surface normal.
How much light is arriving at a surface?
Units of irradiance: Watts/m²
This is a function of incoming angle.
A surface experiencing radiance \( L(x, \theta, \phi) \) coming in from solid angle \( d\omega \) experiences irradiance:

\[
E(x) = L(x, \theta, \phi) \cos \theta d\omega
\]

Crucial property:
Total irradiance arriving at the surface is given by adding irradiance over all incoming angles

Total irradiance is:

\[
\int \int L(x, \theta, \phi) \cos \theta \sin \theta d\theta d\phi
\]

What is image irradiance \( E \) for a radiance \( L \) emitted from a point \( P \)?

Let \( \delta \omega \) be the solid angle subtended by \( \delta A \) (or \( \delta A' \)) from the center of the lens

\[
\delta \omega = \frac{\delta A \cos \alpha}{\left( \delta A \cos \alpha \right)^2 + 4 \left( \frac{\delta A}{2} \right)^2 \cos \alpha}
\]

\[
\delta \omega = \frac{\delta A \cos \alpha}{\left( \delta A \cos \alpha \right)^2 + 4 \left( \frac{\delta A}{2} \right)^2 \cos \alpha}
\]

\[
\frac{\delta A}{\delta A'} \cos \alpha \left( \frac{\delta A}{\delta A'} \right)^{-1}
\]

The power \( \delta P \) emitted from the patch \( \delta A \) with radiance \( L \) and falling on the lens is:

\[
\delta P = \delta A \cos \beta \left( \frac{\pi}{4 \left( \delta A' \right)^2} \right) L \cos \alpha
\]

\[
\delta P = \delta A \cos \beta \left( \frac{\pi}{4 \left( \delta A' \right)^2} \right) L \cos \alpha
\]

\[
E = \left( \frac{\pi}{4 \left( \delta A' \right)^2} \right) \cos \alpha \left( \frac{\delta A}{\delta A'} \right)^{-1}
\]

- \( E \): Image irradiance
- \( L \): emitted radiance
- \( d \): Lens diameter
- \( x' \): depth of image plane
- \( \alpha \): Angle of patch from optical axis