Motion

Computer Vision I
CSE252A
Lecture 17

Announcements

- HW3 is due on Tuesday

Structure from Motion

The change in spatial location between the two cameras (the “motion”)

Locations of points on the object (the “structure”)

MOVING CAMERAS ARE LIKE STEREO

Motion

Some problems of motion
1. Correspondence: Where have elements of the image moved between image frames
2. Reconstruction: Given correspondence, what is the 3-D geometry of the scene
3. Segmentation: What are regions of image corresponding to different moving objects
4. Tracking: Where have objects moved in the image? related to correspondence and segmentation.

Variations:
- Small motion (video),
- Wide-baseline (multi-view)

Is motion estimation inherent in humans?

Demo
Motion

“When objects move at equal speed, those more remote seem to move more slowly.”
- Euclid, 300 BC

The Motion Field

Where in the image did a point move?

Down and left

What causes a motion field?

1. Camera moves (translates, rotates)
2. Object’s in scene move rigidly
3. Objects articulate (pliers, humans, animals)
4. Objects bend and deform (fish)
5. Blowing smoke, clouds

THE MOTION FIELD

The “instantaneous” velocity of all points in an image

The Focus of Expansion (FOE)

Intersection of velocity vector with image plane

With just this information it is possible to calculate:
1. Direction of motion
2. Time to collision

Rigid Motion: General Case

Rigidity Motion:
Velocity Vector: $\dot{P}$
Angular Velocity Vector: $\omega$ (or $\Omega$)

$$\dot{P} = T + \omega \times p$$
General Motion

\[
\begin{bmatrix}
\dot{u} \\
\dot{v}
\end{bmatrix} = \frac{f}{z} \begin{bmatrix}
x \\
y
\end{bmatrix} + \frac{f}{z^2} \begin{bmatrix}
x' \\
y'
\end{bmatrix}
\]

Let \((x, y, z)\) be functions of time \((x(t), y(t), z(t))\):

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{z}
\end{bmatrix} = \frac{f}{z} \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} + \frac{f}{z^2} \begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix}
\]

Substitute \(\dot{p} = T + \omega \times p\) where \(p = (x, y, z)^T\)

Motion Field Equation

\[
\begin{aligned}
\dot{u} &= \frac{T_u - T_z f}{Z} - \omega_z f - \omega_x y - \frac{\omega_y v}{f} \\
\dot{v} &= \frac{T_v - T_z f}{Z} + \omega_z f - \omega_y x - \frac{\omega_x v}{f}
\end{aligned}
\]

- \(T\): Components of 3-D linear motion
- \(\omega\): Angular velocity vector
- \((u, v)\): Image point coordinates
- \(Z\): depth
- \(f\): focal length

Pure Translation

\[\omega = 0\]

\[
\begin{aligned}
\dot{u} &= \frac{T_u - T_z f}{Z} - \omega_z f - \omega_x y - \frac{\omega_y v}{f} \\
\dot{v} &= \frac{T_v - T_z f}{Z} + \omega_z f - \omega_y x - \frac{\omega_x v}{f}
\end{aligned}
\]

Forward Translation & Focus of Expansion

[Giibson, 1950]

Sideways Translation

Parallel (FOE point at infinity)
\(T_z = 0\)
Motion parallel to image plane

Parallel (FOE point at infinity)
\(T_z = 0\)
Motion parallel to image plane
Pure Rotation: $T=0$

$$\begin{align*}
\dot{u} &= \frac{u - T_x f - \omega_z f + \omega_y v + \frac{\omega_z \mu^2}{f} - \frac{\omega_y \nu^2}{f}}{Z} \\
\dot{v} &= \frac{T_x f}{Z} - \frac{v - \omega_z u - \frac{\omega_y \nu^2}{f}}{Z} - \frac{\omega_y \nu^2}{f}
\end{align*}$$

- Independent of $T_x, T_y, T_z$
- Independent of $Z$
- Only function of $(u,v), f$ and $\omega$

Rotational MOTION FIELD

The “instantaneous” velocity of points in an image

Pure Rotation

$$\omega = (0,0,1)^T$$

Motion Field Equation: Estimate Depth

$$\begin{align*}
\dot{u} &= \frac{T_x u - T_y f}{Z} - \omega_z f + \omega_y v + \frac{\omega_z \mu^2}{f} - \frac{\omega_y \nu^2}{f} \\
\dot{v} &= \frac{T_y v - T_x f}{Z} + \omega_z f - \omega_y u - \frac{\omega_y \nu^2}{f} - \frac{\omega_y \nu^2}{f}
\end{align*}$$

If $T, \omega_0$ and $f$ are known or measured, then for each image point $(u,v)$, one can solve for the depth $Z$ given measured motion $(du/dt, dv/dt)$ at $(u,v)$.

Estimating the motion field from images

1. Feature-based (Sect. 8.4.2 of Trucco & Verri)
   1. Detect Features (corners) in an image
   2. Search for the same features nearby (Feature tracking).
2. Differential techniques (Sect. 8.4.1)

Definition of optical flow

**OPTICAL FLOW** = apparent motion of brightness patterns

Ideally, the optical flow is the projection of the three-dimensional velocity vectors on the image.
Mathematical formulation

\( I(x,y,t) \) = brightness at image point \((x,y)\) at time \(t\)

Consider scene (or camera) to be moving, so \((x,y)\) is a function of time (i.e., \(x(t), y(t)\)), and point is moving with velocity \((dx/dt, dy/dt)\)

Brightness constancy assumption:

\[ I(x + dx/dt, y + dy/dt, t + dt) = I(x, y, t) \Rightarrow \frac{dI}{dt} = 0 \]

Optical flow constraint equation:

\[ \frac{dI}{dt} = \frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0 \]

Solving for flow

Optical flow constraint equation:

\[ \frac{dI}{dt} = \frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0 \]

- We can measure \(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}, \frac{\partial I}{\partial t}\)
- We want to solve for \(\frac{dx}{dt}, \frac{dy}{dt}\)
- One equation, two unknowns

Aperture Problem and Normal Flow

Measurements

\[ I_x, I_y, I_t \]

Flow vector

\[ u = \frac{dx}{dt}, \quad v = \frac{dy}{dt} \]

The component of the optical flow in the direction of the image gradient.

Normal Flow

Illusion Works Barber Pole Illusion

What is the correspondence of \(P\) & \(P'\)

Contour plots of image intensity in two images
Two ways to get flow

1. Think globally, and regularize over image
2. Look over window and assume constant motion in the window

Horn & Schunck algorithm

Additional smoothness constraint:

\[ e_c = \iint (u_x^2 + u_y^2 + v_x^2 + v_y^2) \, dx \, dy, \]

besides OF constraint equation term

\[ e_\text{OF} = \iint (I_x u + I_y v + I_t)^2 \, dx \, dy, \]

minimize \[ e_\text{OF} + \lambda e_c \]

Lucas-Kanade: Integrate over a Patch

Assume a single velocity for all pixels within an image patch

\[ E(u, v) = \sum_{(x,y)} (I_x(x,y)u + I_y(x,y)v + I_t)^2 \]

\[ \frac{dE(u, v)}{du} = 2 \iint (I_x(x,y)u + I_y(x,y)v + I_t) \, dx \, dy = 0 \]

\[ \frac{dE(u, v)}{dv} = 2 \iint (I_x(x,y)u + I_y(x,y)v + I_t) \, dx \, dy = 0 \]

Solve with:

\[ \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix} \]

On the LHS: sum of the 2x2 outer product tensor of the gradient vector

\[ (\nabla I^T \nabla I) \vec{u} = -\nabla I \]

Lucas-Kanade: Singularities and the Aperture Problem

Let \[ M = \sum |V|^2 \] and \[ b = \left[ -\sum I_x I_t \right] \]

- Algorithm: At each pixel compute \[ U \] by solving \[ MU = b \]
- \[ M \] is singular if all gradient vectors point in the same direction (e.g., along an edge)
- of course, trivially singular if the summation is over a single pixel (i.e., only normal flow is available (aperture problem))
- Corners and textured areas are OK