Problem 1 Show that $\leq_m$ is reflexive and transitive; give a counterexample showing that $\leq_m$ is not symmetric.

Problem 2 Let $B$ be a decidable language such that the string “aaa” is in $B$ but the string “bbb” is not in $B$. Show that a language $A$ is decidable if and only if $A \leq_m B$.

Problem 3 Let $L$ be the language

$$\{ (M_1, M_2) \mid M_1 \text{ and } M_2 \text{ are Turing machines and } L(M_1) \leq_m L(M_2) \}.$$ 

Show that $L$ is undecidable.

Problem 4 (Sipser 5.22) Show that a language $L$ is R.E. (i.e., Turing-recognizable) if and only if $L \leq_m A_{TM}$ (where $A_{TM}$ is the acceptance problem for Turing machines).