

Homework #7

Due: Thursday, December 3rd, 2009.

Problem 1 Show that \leq_m is reflexive and transitive; give a counterexample showing that \leq_m is not symmetric.

Problem 2 Let B be a decidable language such that the string “aaa” is in B but the string “bbb” is not in B . Show that a language A is decidable if and only if $A \leq_m B$.

Problem 3 Let L be the language

$$\{ \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines and } L(M_1) \leq_m L(M_2) \} .$$

Show that L is undecidable.

Problem 4 (Sipser 5.22) Show that a language L is R.E. (i.e., Turing-recognizable) if and only if $L \leq_m A_{\text{TM}}$ (where A_{TM} is the acceptance problem for Turing machines).