Problem 1 Let \( Z_{DFA} \) be the language
\[
\{ \langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ includes all strings of the form } 0(0 \cup 1)^* \}. 
\]
Show that \( Z_{DFA} \) is decidable.

Problem 2 \( NOSYMB \) be the language
\[
\{ \langle M, w, t \rangle \mid M \text{ is a Turing machine, } w \text{ is a string, and } t \text{ is a symbol; and } M, \text{ when run on input } w, \text{ never writes the symbol } t \}.
\]
Show that \( NOSYMB \) is undecidable.

Hint: Assume that \( NOSYMB \) is decidable. You’ll use this to decide the halting problem, yielding a contradiction. Given the description \( \langle M, w \rangle \) of a Turing machine \( M \) and string \( w \), use \( M \) and \( w \) to devise a Turing machine \( M' \), a string \( w' \), and a symbol \( t' \); then run the decider for \( NOSYMB \) on \( \langle M', w', t' \rangle \) and use its answer to decide whether \( M \) accepts \( w \) or not.

Problem 3 Let \( BOB_{DFA} \) be the language
\[
\{ \langle A \rangle \mid A \text{ is a DFA and there exists some palindrome } w \text{ such that } w \in L(A) \}.
\]
Show that \( BOB_{DFA} \) is decidable.

Hint: Recall the result in problem 4 of midterm 2: If \( L_1 \) is regular and \( L_2 \) is context-free, then \( L_1 \cap L_2 \) is context-free. For the definition of palindromes, see problem 1 of homework 3.

Problem 4 Let \( LEN_{TM} \) be the language
\[
\{ \langle M, k \rangle \mid M \text{ is a Turing machine, } k \text{ an integer, and there exists a string } w \text{ such that } w \text{ is of length } k \text{ and } M \text{ accepts } w \text{ in at most } k \text{ steps} \}.
\]
Show that \( LEN_{TM} \) is decidable.

Problem 5 Prove that the class of recursive (i.e., decidable) languages is closed under Kleene star.