

1. How many different arrangements can be formed using **all** the letters in the word SCHWARZENEGGER?
2. Prove by induction that for  $n \geq 1$ ,

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

3. An urn contains 2 red balls, 3 white balls and 4 blue balls. You reach into the urn and randomly remove three balls from the urn (**without** replacement). What is the probability that the balls removed all have **different** colors?

- (a)  $\frac{1}{7}$
- (b)  $\frac{2}{7}$
- (c)  $\frac{1}{2}$
- (d)  $\frac{3}{7}$
- (e) none of the above.

4. 5 cards are selected at random from an ordinary deck of 52 cards. What is the probability that **at least two** of the cards are Aces?

5. In how many ways can 20 cookies and 30 jellybeans be distributed to 4 boys and 4 girls if every boy is to get at least 2 cookies, and every girl is to get at least 1 cookie and at least 3 jellybeans?

6. Two urns  $U_1$  and  $U_2$  each contain red and blue balls.  $U_1$  contains 3 red and 4 blue balls.  $U_2$  contains 4 red and 5 blue balls. A random urn is selected and three balls are removed. What is the expected number of blue balls that are removed?

7. Eight light bulbs are selected at random from a set of 20 light bulbs of which 4 are defective. What is the probability that **exactly two** of the selected bulbs are defective?

8. Show that the alternating sum

$$\sum_{k=0}^n (-1)^k \binom{n}{k} = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} \dots = 0$$

for all  $n \geq 1$ .

9. A person has 2 coins. One is a fair coin with a heads and a tails on the two sides. The other is a 2-headed coin, i.e, both sides are heads. One of these two coins is selected at random and flipped. If this coin comes up heads then it is exchanged with the other coin. However, if this first flip is tails then the coin is not exchanged. Now the current coin is flipped. If this flip comes up heads, then as before, the coin is exchanged with the other coin (and if tails, then it is not exchanged). Finally, this last coin is flipped once more.

- (a) What is the probability that the final flip is a heads?
- (b) What is the probability that the second flip was tails given that the last flip was heads?