Problem 1: UCSD Recreation Activities Information Session (URAIS)

As part of the UCSD sports tradition, at the beginning of each year, an information session is organized for new incoming students. The organizers, in an effort to make this session as informative as possible, have managed to recruit \( n \) older students to talk about the various activities they have participated in and share their experiences with the incoming students. Students will hold a panel discussion where they will try to answer the questions from the incoming students regarding the available activities. Since the audience’s interest are likely to vary, the organizers ideally want to have at least one representative from each of the \( m \) activities. However, due to the large number of incoming students this year, the organizers will need to hold two parallel info sessions to accommodate all incoming students. The organizers have a list with the \( m \) activities and the names of participants (among those \( n \) volunteers) for each one of the activities. (The same volunteer may be listed under multiple activities.) They like to decide whether they can divide the \( n \) volunteers into two groups such that in both info sessions there are representatives (from the group of the \( n \) volunteers) from all \( m \) activities.

- Formulate the above problem as a decision problem in the language of sets and give the correspondence between the parameters of the decision problem in the formal description and the entities in the problem. Let URAIS denote this decision problem.
- Give (in pseudocode) a polynomial time (map) reduction from a known NP-complete problem to URAIS. As a starting point of your reduction you can use any NP-Complete problem presented in class, in the textbook or in previous homeworks.
- Prove the correctness of your reduction.
- Conclude that URAIS is NP-Complete.

Problem 2: EXP completeness

The best known solution to NP complete problems seems to require enumeration and take exponential time. A natural question to ask is whether all exponential time computations can be performed in NP. (In class we remarked how this, if true, would yield a proof that \( P \neq NP \) by the time hierarchy theorem.) This is the NP=EXP question, a long standing open problem in computer science. In this problem you use the notion of completeness to set up a framework for the study of this question, similar to what we have done for the P=NP question.

The parts of this problem are not independent. E.g., you can give a trivial definition for part (1) which makes part (2) trivial, but then you will have trouble providing a solution to part (3).

1. Give a definition of EXP-complete problem. You definition should read something like “A language \( A \) is EXP-complete if ...”
2. Give an example of EXP-complete language \( C \): formally define \( C \) as a set of lists, and prove that your proposed problem is indeed EXP-complete according to your definition
3. Prove that if \( C \) could be solved in NP, then NP=EXP.