Problem 1: Describing a TM. Let $\Sigma = \{-, a, b\}$ and 
$$L = \{w-w-w \mid w \in \{a, b\}^*\}.$$ 
We saw on the previous homework that this problem is not context-free.

a.) Give a high level description of a turing machine $M$ that decides $L$. The level of detail should be similar to level in Sipser Example 3.7.

b.) Draw a state diagram for $M$. You may assume $M$ has a doubly infinite tape.
Problem 2. Binary Writer Competition. This is a homework problem and a competition. A binary writer is a TM with a doubly infinite tape (see Sipser problem 3.11 if you don’t know what this is) and tape alphabet $\Gamma = \{0, 1, \sqcup\}$ that, when started with a completely blank tape (all $\sqcup$), terminates in a finite number of steps with a binary number written somewhere on the tape. In this problem, we will be concerned with binary writers with only three states $q_0, q_1, q_h$, where $q_0$ is the start state and $q_h$ is the halt state; as soon as the machine enters $q_h$ it halts.

An example 3-state binary writer is given below. It terminates with the value $11 = 3$ on the tape.

![Binary Writer Diagram]

For this problem, your goal is to give a 3-state binary writer that halts with the largest possible number on the tape. The student with the binary writer that halts with the largest number on the tape will win the competition. To get 8 points on this problem you should get at least 15, and anything over 15 will get 10. If you want to enter the competition, you should use the program JFLAP (there is a link on the course webpage) and email Scott your binary writer. It is highly recommended that everyone (including those who do not want to enter the competition) use JFLAP to test their binary writer and try to improve it.

Note: You should assume that the TM has a doubly-infinite tape and that the TM must move the head left or right at each transition (i.e., it cannot stay on the same tape cell).
Problem 3. Variations on TMs. For any number \( n \) and set \( S \), let \( \sigma_n(S) \) be the set of all sequences of length \( n \) such that each element of the sequence is in \( S \). For example, if \( S = \{1, 2, 3\} \) then \( \sigma_2(S) = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\} \).

An \( n \)-at-a-time TM is a Turing machine with transition function

\[
\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \sigma_n(\{L,R\}).
\]

The machine will thus move the head \( n \) times for every transition. For example, if \( n = 3 \) and \( \delta(q, a) = (r, b, (L,L,L)) \), when in state \( q \) and reading an \( a \), the machine writes \( b \) on the tape, goes to state \( r \), and moves 3 spaces to the left. Note that a 1-at-a-time TM is the same as a regular TM.

a.) Show that 3-at-a-time TMs recognize the class of Turing-recognizable languages.

b.) Show that 2-at-a-time TMs do not recognize the class of Turing-recognizable languages.

Note: Again, you should assume the TM has a doubly-infinite tape.
Problem 4. Closure. Show that the collection of decidable languages is closed under the following operations:

a.) Intersection

b.) Concatenation

Problem 5. More Closure. Find the flaw in the following proof that the class of Turing-recognizable languages is closed under complement.

Let $L$ be some Turing-recognizable language. Since $L$ is recognizable, there is some Turing machine $M$ that recognizes it. We show that $\overline{L}$ is recognizable by giving a TM $M'$ that recognizes it:

$$ M' = \text{“On input } x. $$

1.) Run $M$ on input $x$.

2.) If $M$ accepts $x$, then reject, otherwise accept.”

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