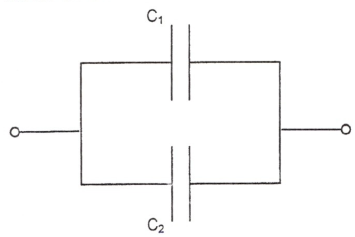
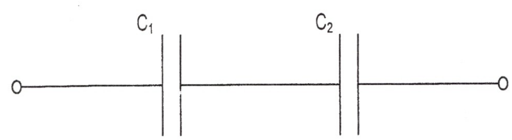
ECE 53A Fall 2007, Midterm 3, November 27, 2007, Name SOLUTIONS

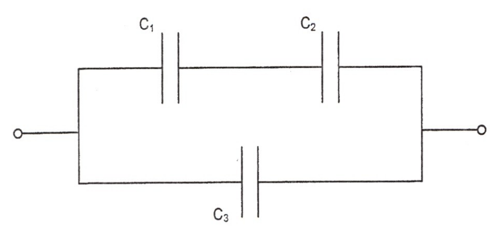
1. Find the equivalent capacitance between the two terminals in each of the three networks.



$C_1 + C_2$



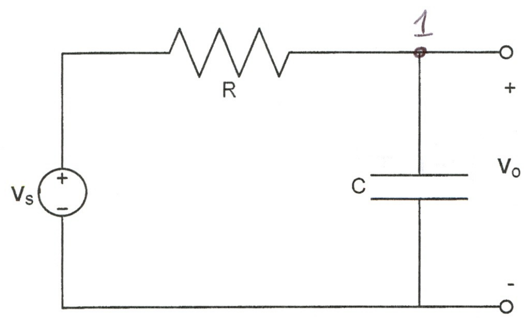
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} + C_3$$

$$= \frac{C_1 C_2 + C_2 C_3 + C_1 C_3}{C_1 + C_2}$$

2. In the following circuit, the voltage source v_s is a step function u(t), i.e. u(t)= 0 for t<0, u(t)=1 for t>= 0. At time t=0, output voltage v_o= 0. Derive the function v_o(t) for t>= 0.



Using KCL at node 1,

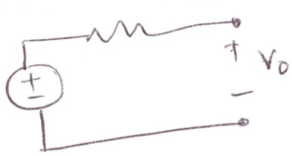
$$\frac{V_s - V_o}{R} = C\frac{dV_o}{dt}$$

$$CR\frac{dV_o}{dt} + V_o = V_s$$

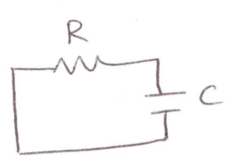
$$\therefore V_o = V_s\left(1 - e^{-t/CR}\right) = 1\left(1 - e^{-t/CR}\right), \text{ when } t \geq 0$$

Method 2

$$V_o = b\left(1 - e^{-t/a}\right)$$

$b \Rightarrow$ 

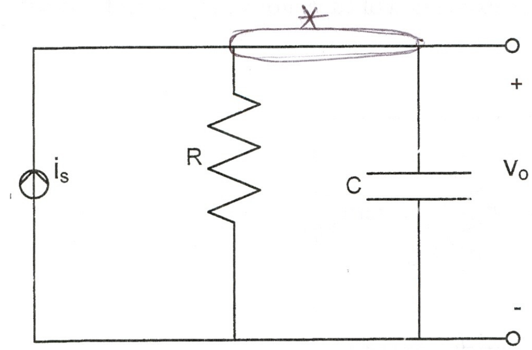
$V_o = V_s$

$a =$ 

$a = CR$

$$\therefore V_o = V_s\left(1 - e^{-t/CR}\right)$$

3. In the following circuit, the current source i_s is a step function u(t), i.e. u(t)= 0 for t<0, u(t)=1 for t>= 0. At time t=0, output voltage v_o= 0. Derive the function $v_o(t)$ for t>= 0.



Applying KCL at node *,

$$i_s = \frac{V_o}{R} + C\frac{dV_o}{dt}$$

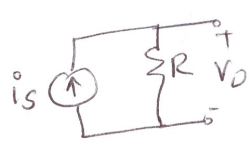
$$i_s R = V_o + CR\frac{dV_o}{dt}$$

$$\therefore V_o = i_s R(1-e^{-t/CR}), \quad \text{when } t \geq 0$$

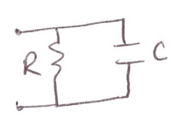
$$= R(1-e^{-t/CR}) \because \text{when } t \geq 0,\ i_s = 1A.$$

Method 2

$b = i_s R$

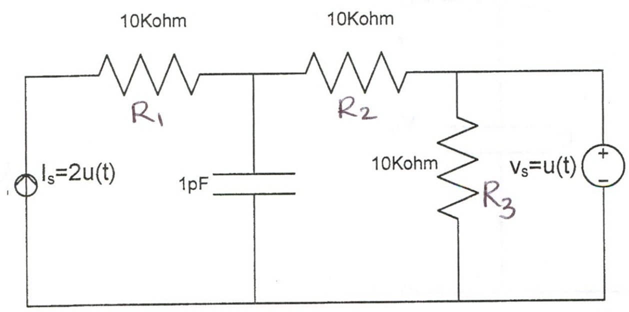


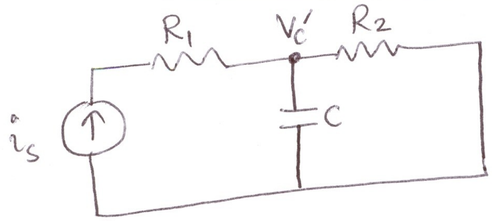
$a = RC$

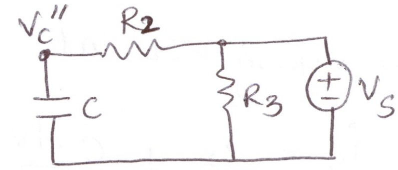


$$\therefore V_o = i_s R(1-e^{-t/CR})$$
$$= R(1-e^{-t/CR})$$

4. In the following circuit, the voltage source $v_s = u(t)$, the current source $i_s = 2u(t)$, where $u(t)$ is a step function, i.e. $u(t) = 0$ for $t<0$, $u(t) = 1$ for $t >= 0$. Initially, the voltage accross the capacitor is reset, i.e. $v_C = 0$. Find an expression of capacitor voltage $v_C(t)$ for $t >= 0$.







$$C\frac{dV_c'}{dt} + \frac{V_c'}{R_2} = i_s \rightarrow (1) \qquad C\frac{dV_c''}{dt} = \frac{V_s - V_c''}{R_2} \rightarrow (2)$$

(1) + (2) (Super position principle)

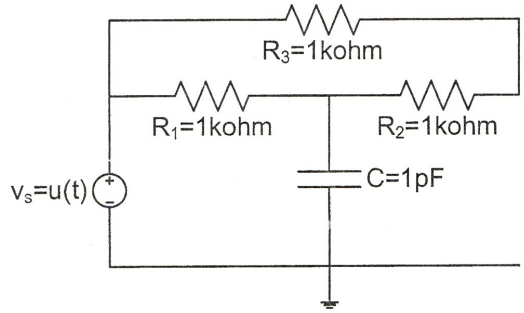
$$C\left(\frac{dV_c'}{dt} + \frac{dV_c''}{dt}\right) + \frac{V_c'}{R_2} = i_s + \frac{V_s}{R_2} - \frac{V_c''}{R_2}$$

$$C\frac{dV_c}{dt} + \frac{V_c}{R_2} = i_s + \frac{V_s}{R_2} \qquad \left(\text{because } \frac{dV_c'}{dt} + \frac{dV_c''}{dt} = \frac{d(V_c' + V_c'')}{dt}\right)$$

$$CR_2\frac{dV_c}{dt} + V_c = i_s R_2 + V_s$$

$$\Rightarrow V_c = (i_s R_2 + V_s)\left(1 - e^{-t/CR_2}\right) = \underline{\underline{(20\,kV + 1V)\left(1 - e^{-t/10ns}\right)}}$$

5. Given a RC-network shown below, the voltage source v_s= u(t) is a step input, i.e. u(t)= 0 for t< 0, u(t)= 1 for t>= 0.



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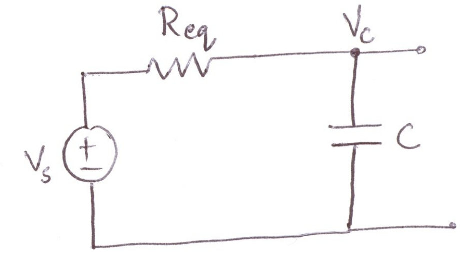

a. Derive the differential equation for the capacitor voltage, $v_C(t)$, given the initial condition that $v_C(0)$= 0V.

$$R_{eq} = (R_2 + R_3) \| R_1 = \frac{2\,k\Omega * 1\,k\Omega}{3\,k\Omega} = \frac{2}{3}\,k\Omega$$

$$\frac{V_s - V_c}{R_{eq}} = C\frac{dV_c}{dt} \Rightarrow C\,R_{eq}\frac{dV_c}{dt} + V_c = V_s$$

$$\left(\frac{2}{3}\times 10^{-9}\right)\frac{dV_c}{dt} + V_c = 1$$

b. Solve for capacitor voltage $v_C(t)$. What is the time constant for this network?

$$V_c(t) = V_s\left(1 - e^{-t/\tau}\right)$$

$$= 1\left(1 - e^{-t/\left(\frac{2}{3}\times 10^{-9}\right)}\right)\ V$$

Time constant, $\tau = \frac{2}{3}$ ns